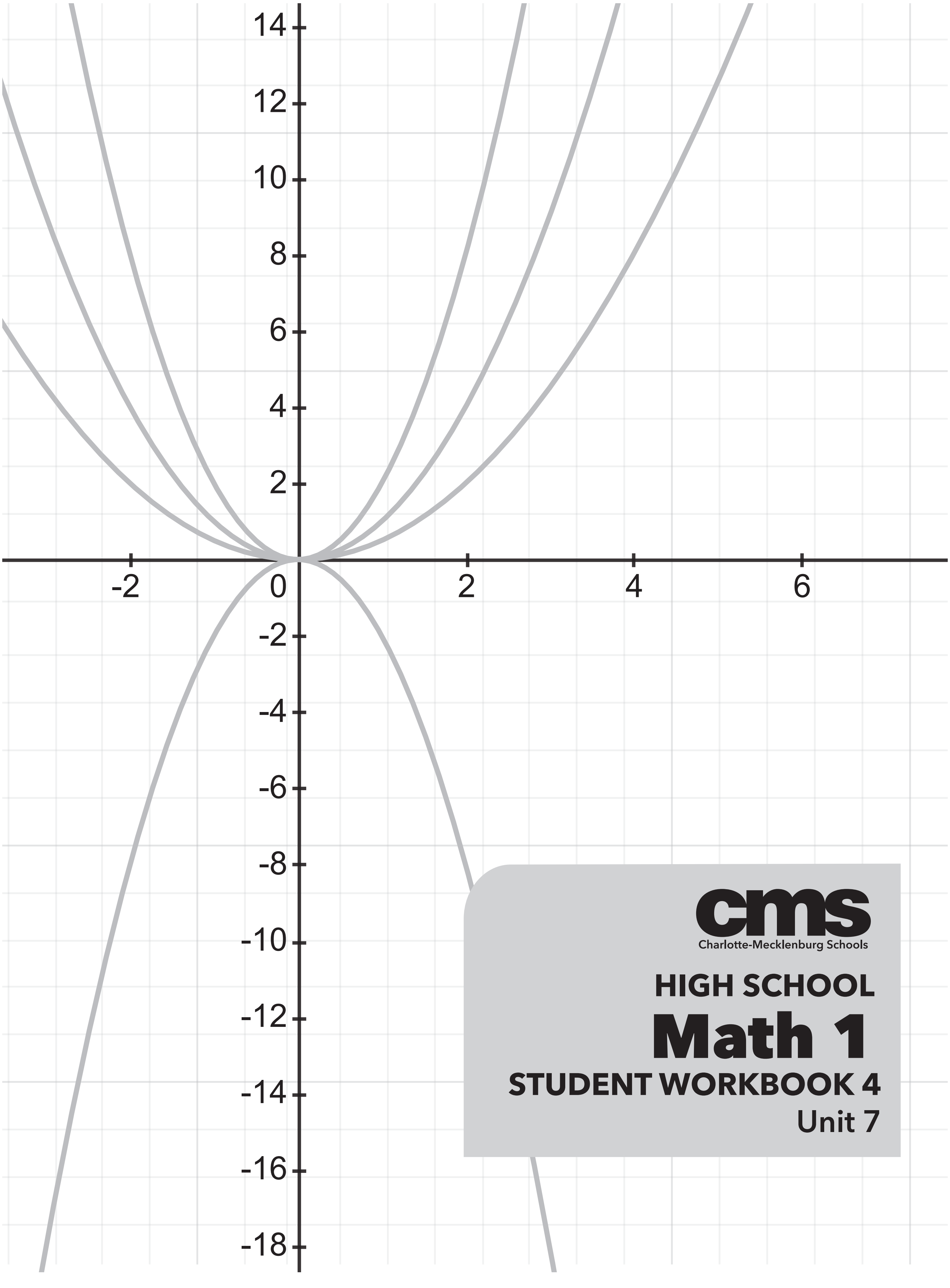

Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

STUDENT WORKBOOK 4

Unit 7

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Unit 7 Quadratic Functions and Equations

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Lesson 1: A Different Kind of Change

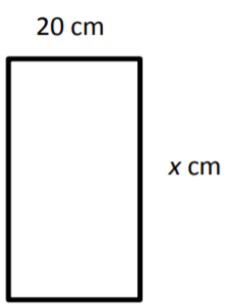
Learning Targets

- I can describe a pattern of change represented by a graph that is not linear or exponential.
- I can create drawings, tables, and graphs that represent the area of a garden.

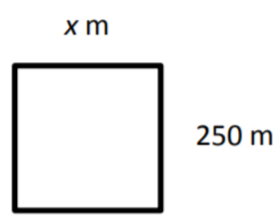
Bridge

Given the rectangle's perimeter, find the unknown side length and the area.[1]

a. P = 120 cm



b. P = 1,000 m



Warm-up: Three Tables

Look at the patterns in the three tables. What do you notice? What do you wonder?

x	y
1	0
2	5
3	10
4	15
5	20

x	y
1	3
2	6
3	12
4	24
5	48

x	y
1	8
2	11
3	10
4	5
5	-4

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Activity 1: Measuring a Garden

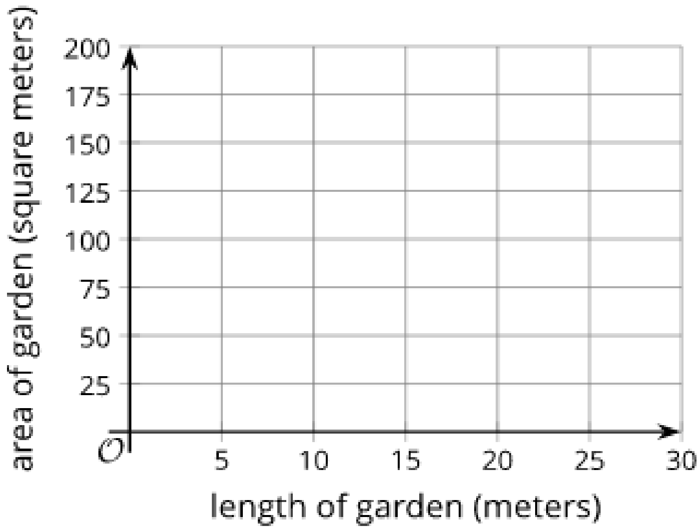
Noah has 50 meters of fencing to completely enclose a rectangular garden in the backyard.

1. Draw some possible diagrams of Noah’s garden. Label the length and width of each rectangle.

2. Find the length and width of such a rectangle that would produce the largest possible area. Explain or show why you think that pair of length and width gives the largest possible area.

Activity 2: Plotting the Measurements of the Garden

1. Plot some values for the length and area of the garden on the coordinate plane.



2. What do you notice about the plotted points?

3. The points $(3, 66)$ and $(22, 66)$ each represent the length and area of the garden. Plot these two points on coordinate plane, if you haven't already done so. What do these points mean in this situation?

Are You Ready For More?

1. Find a few other pairs of points representing (length, area), like $(3, 66)$ and $(22, 66)$, that have different x-coordinates but the same y-coordinate.

2. What do you notice about all these pairs of points? How would you explain to a friend how to find more?

Lesson Debrief

Lesson 1 Summary and Glossary

In this lesson, we looked at the relationship between the side length and the area of a rectangle when the perimeter is unchanged.

If a rectangle has a perimeter of 40 inches, we can represent possible lengths and widths as shown in the table.

- We could also consider lengths and widths that are decimal values, such as 6.5 inches and 13.5 inches. For the purpose of looking at the relationship between length, width, and area, we have chosen to look at the whole-number lengths and widths.
- We know that twice the length and twice the width must equal 40, which means that the length plus width must equal 20.

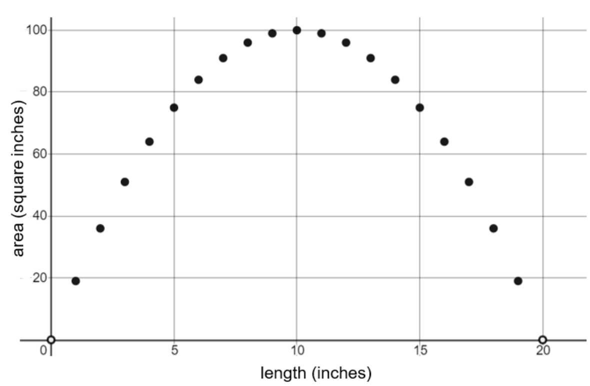
What about the relationship between the side lengths and the area of rectangles with a perimeter of 40 inches?

- Here are the areas of some different rectangles whose perimeter are 40 inches.

Length (inches)	Width (inches)	Area (square inches)
1	19	19
2	18	36
3	17	51
4	16	64
5	15	75
6	14	84
7	13	91
8	12	96
9	11	99
10	10	100

Length (inches)	Width (inches)	Area (square inches)
11	9	99
12	8	96
13	7	91
14	6	84
15	5	75
16	4	64
17	3	51
18	2	36
19	1	19

- Here is a graph of the lengths and areas represented in the table:



A few things to notice about the relationship shown in the table and the graph:

- The length cannot be 0 inches because a rectangle cannot have zero length. The length also cannot be 20 inches because that would make the width of the rectangle zero. (These points have been plotted with an open circle.) The length must be more than 0 inches and less than 20 inches.
- Initially, as the length of the rectangle increases from 0 inches to 10 inches, the area also increases. Later, however, as the length increases from 10 inches to 20 inches, the area decreases.
- The highest point on the graph is (10,100). This means the maximum area of the rectangle occurs when the length of the rectangle is 10 inches and the area is 100 square inches.

We have not studied relationships like this yet and will investigate them further in this unit.

Unit 7 Lesson 1 Practice Problems

1. Here are a few pairs of positive numbers whose sums are 50.

   a. Find the product of each pair of numbers.

First number	Second number	Product
1	49	
2	48	
10	40	

   b. Find a pair of numbers that has a sum of 50 and will produce the largest possible product.

   c. Explain how you determined which pair of numbers has the largest product.

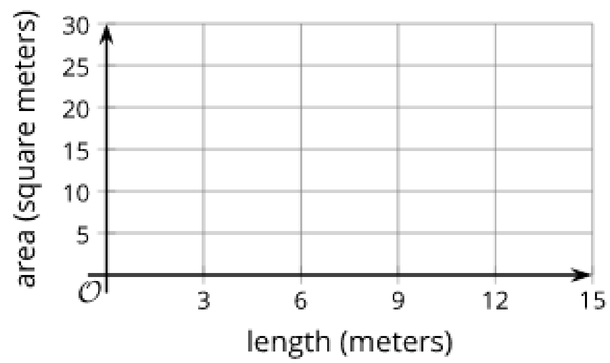
2. Here are some lengths and widths of several rectangles whose perimeters are 20 meters.

   a. Complete the table. What do you notice about the areas?

   b. Without calculating, predict whether the area of the rectangle will be greater or less than 25 square meters if the length is 5.25 meters.

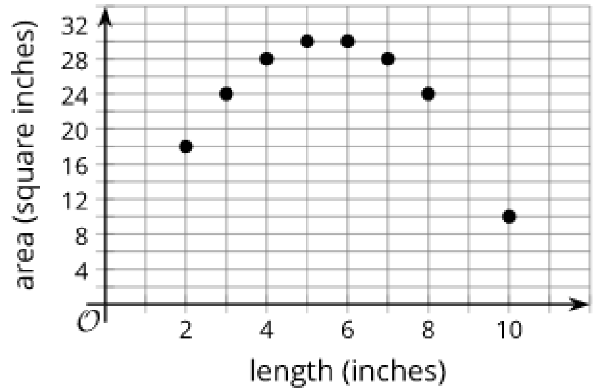
Length (meters)	Width (meters)	Area (square meters)
1	9	
3	7	
5		
7		
9		

   c. On the coordinate plane, plot the points for length and area from your table.



   d. Do the values change in a linear way? Do they change in an exponential way?

3. Which statement best describes the relationship between a rectangle's side length and area as represented by the graph?



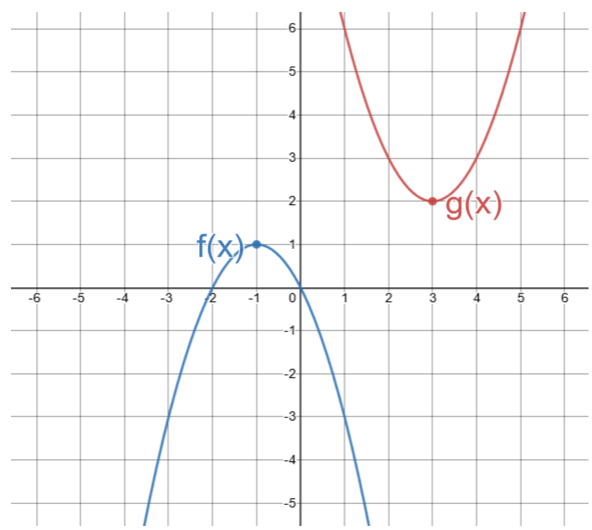
   a. As the side length increases by 1, the area increases and then decreases by an equal amount.

   b. As the side length increases by 1, the area increases and then decreases by an equal factor.

   c. As the side length increases by 1, the area does not increase or decrease by an equal amount.

   d. As the side length increases by 1, the area does not change.

4. The graph shows two functions, $f(x)$ and $g(x)$. For each function:



   a. On what interval is the value increasing for each function?

   b. On what interval is the value decreasing for each function?

5. Copies of a book are arranged in a stack. Each copy of a book is 2.1 cm thick.
    a. Complete the table.
    b. What do you notice about the differences in the height of the stack of books when a new copy of the book is added?
    c. What do you notice about the factor by which the height of the stack of books changes when a new copy is added?

Copies of book	Stack height in cm
0	
1	
2	
3	
4	

    d. How high is a stack of b books?

(From Unit 6)

6. The value of a phone when it was purchased was $500. It loses $\frac{1}{5}$ of its value a year.
    a. What is the value of the phone after 1 year? What about after 2 years? 3 years?
    b. Tyler says that the value of the phone decreases by $100 each year since $\frac{1}{5}$ of 500 is 100. Do you agree with Tyler? Explain your reasoning.

(From Unit 6)

7. (*Technology required.*) The data in the table represent the price of one gallon of milk in different years.

   Use graphing technology to create a scatter plot of the data.

x, time (years)	Price per gallon of milk (dollars)
1930	0.26
1935	0.47
1940	0.52
1940	0.50
1945	0.63
1950	0.83
1955	0.93
1960	1.00
1965	1.05
1970	1.32
1970	1.25
1975	1.57
1985	2.20
1995	2.50
2005	3.20
2018	2.90
2018	3.25

   a. Does a linear model seem appropriate for the data? Why or why not?

   b. If the linear model seems appropriate, create the line of best fit. Round to two decimal places.

   c. What is the slope of the line of best fit, and what does it mean in this context? Is it realistic?

   d. What is the y-intercept of the line of best fit, and what does it mean in this context? Is it realistic?

(From Unit 4)

8. Give a value for r, the correlation coefficient, that indicates that a line of best fit has a negative slope and the scatterplot shows a strong linear relationship.

(From Unit 4)

9. Match each inequality to the graph of its solution.

Inequalities

a. $3x + 4y \leq 36$

b. $12x + 3y \leq 36$

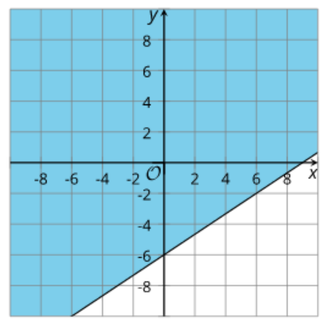
c. $6x + 4y \geq 36$

d. $3x - 9y \geq 36$

e. $4x - 6y \leq 36$

Graphs

1.



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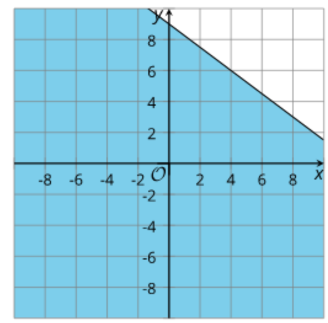

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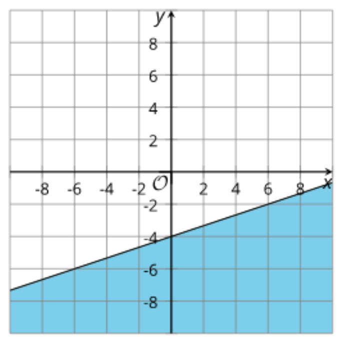

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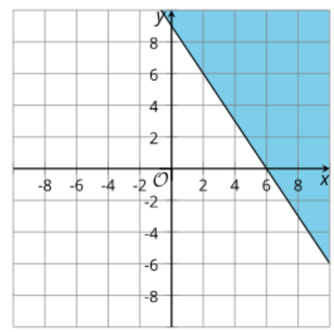

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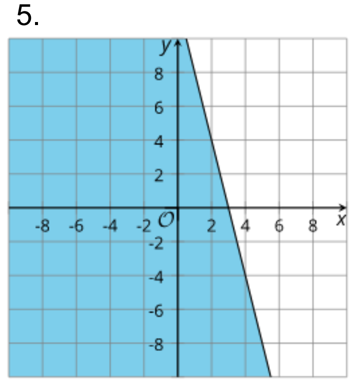


(From Unit 3)

Lesson 2: Expressing Revenue

Learning Targets

- I can identify a quadratic expression.
- I can write and graph a quadratic function to model economic situations.

Bridge

1. Select all expressions that are equivalent to $10 - 2x$.

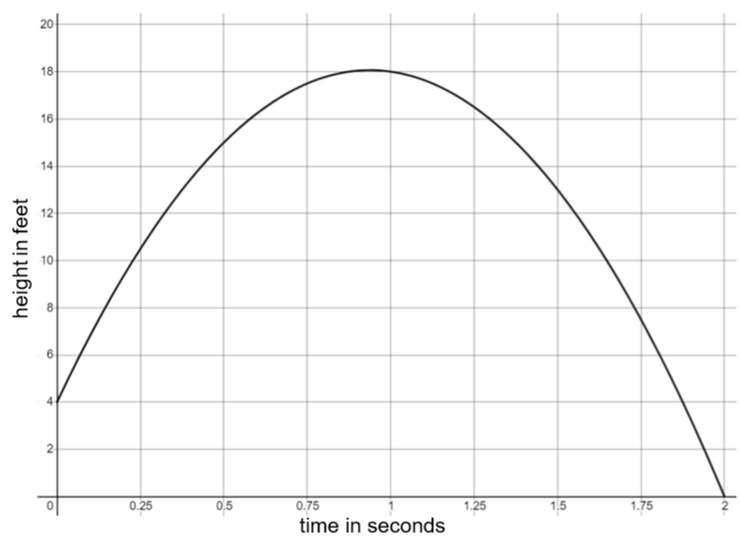
   $2(5 - x)$ $\quad$ $10(1 - 2x)$ $\quad$ $-2(5 - x)$ $\quad$ $-2(x - 5)$

2. Write an equivalent expression for one of the expressions above that is not equivalent to $10 - 2x$.

Warm-up: A Flying Arrow

Clare was learning how to use a bow and arrow. The graph shows the height of the arrow as a function of the time since she released it.

Use the graph to identify each of the following. Be prepared to share your reasoning.



a. At what height did Clare release the arrow?

b. What was the maximum height of the arrow?

c. When did the arrow hit the ground?

Activity 1: What Price to Charge?

A company that sells movies online is deciding how much to charge customers to download a new movie.[1] Based on data from previous sales, the company predicts that if they charge x dollars for each download, then the number of downloads, in thousands, is $18 - x$.

1. Complete the table to show the predicted number of downloads at each listed price. Then, find the revenue at each price. The first row has been completed for you.

Price (dollars per download)	Number of downloads (thousands)	Revenue (thousands of dollars)
3	15	45
5		
10		
12		
15		
18		
x		

2. Write an equation for the revenue, r, as a function of the price per download, x.

3. Verify that $(14, 56)$ is a solution to the equation. Show your work and explain what this means for the graph of the equation.

[1] Adapted from IM 9–12 Math https://curriculum.illustrativemathematics.org/HS/teachers/index.html,

Activity 2: Analyzing Price and Revenue[2]

1. Graph your equation for revenue based on the price for a download using graphing technology.

2. Identify the horizontal intercepts and explain what they mean in this situation.

3. What price would you recommend the company charge for a new movie? Explain your reasoning.

Are You Ready For More?

A function that predicts how much of a product will sell given its price is called a "demand function." An example is the function that uses the price (in dollars per download), x, to determine the number of downloads (in thousands), $18 - x$. Economists are interested in factors that can affect the demand function and therefore the price suppliers wish to set.[3]

1. What are some things that could increase the number of downloads predicted for a given price?

2. If the demand shifted so that we predicted $20 - x$ thousand downloads at a price of x dollars per download, what do you think will happen to the price that gives the maximum revenue? Check what actually happens.

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[2] Adapted from IM 9–12 Math https://curriculum.illustrativemathematics.org/HS/teachers/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.
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Lesson Debrief

Lesson 2 Summary and Glossary

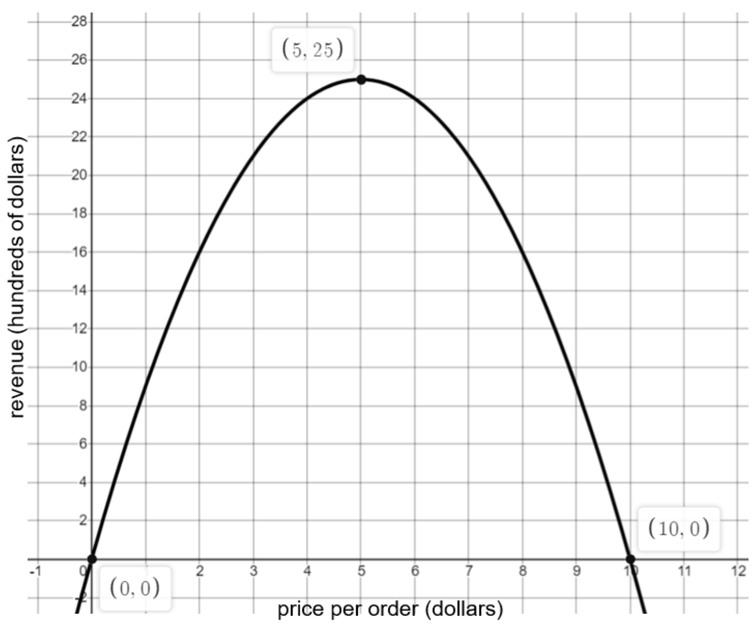
In this lesson, we explored different representations of a quadratic function.

A company is deciding how much to charge customers per order for their meal delivery service. If they charge x dollars per order, then the number of orders, in hundreds, is predicted to be $10 - x$.

Here is a table of values for a sample of prices, orders, and revenue (amount of money earned):

Price (dollars per order)	Number of orders (hundreds)	Revenue (hundreds of dollars)
0	10	0
1	9	9
2	8	16
3	7	21
4	6	24
5	5	25
6	4	24
7	3	21
8	2	16
9	1	9
10	0	0

- We multiply the price per order (x) and the number of orders $(10 - x)$ to calculate the revenue. This gives us the expression $x(10 - x)$, which is equivalent to $10x - x^2$. There is a squared term, so $10x - x^2$ is a quadratic expression.
- The expression can be used to create an equation that gives the revenue, r, as a function of the price, x. That equation is $r = 10x - x^2$.
- The graph of the equation is shown to the right.
- Using the graph, we can see that the horizontal intercepts are (0,0) and (10,0). This means that if the price per order is $0 or $10, the company will make $0 in revenue. We can also see the maximum point (5,25). This means that for a price per order of $5, the maximum revenue is 25 hundred dollars.



Quadratic expression: An expression that can be written in the form $ax^2 + bx + c$ when $a \neq 0$.

Quadratic function: A function where the output is given by a quadratic expression in the input.

Unit 7 Lesson 2 Practice Problems

1. Identify **all** of the quadratic expressions.

x^2	$3x+2$	$4x^2-8$	$3(1.25)^x$
2^x	$-5x^2+7x-1$	$x(x+4)$	$2x$

2. (*Technology required.*) Graph the equation $-x^2+8x$ using technology.

   a. Identify the horizontal intercepts.

   b. Identify the maximum point.

3. Write an expression equivalent to $x(15-x)$ and explain why it is a quadratic expression.

4. Here are a few pairs of positive numbers whose sums are 26.

First number	Second number	Product
2	24	
6	20	
10	16	

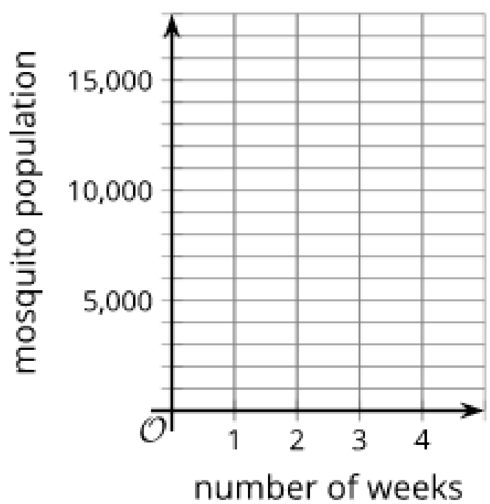
   a. Find the product of each pair of numbers.

   b. Find a pair of numbers that has a sum of 26 and will produce the largest possible product.

   c. Explain how you determined which pair of numbers has the largest product.

(From Unit 7, Lesson 1)

5. A population of mosquitos p is modeled by the equation $p = 1,000 \cdot 2^w$, where w is the number of weeks after the population was first measured.



   a. Find and plot the mosquito population for $w = 0, 1, 2, 3, 4$.

   b. Where on the graph do you see the 1,000 from the equation for p?

   c. Where on the graph can you see the 2 from the equation?

(From Unit 6)

6. The number of copies of a book sold the year it was released was 600,000. Each year after that, the number of copies sold decreased by $\frac{1}{2}$.

   a. Complete the table showing the number of copies of the book sold each year.

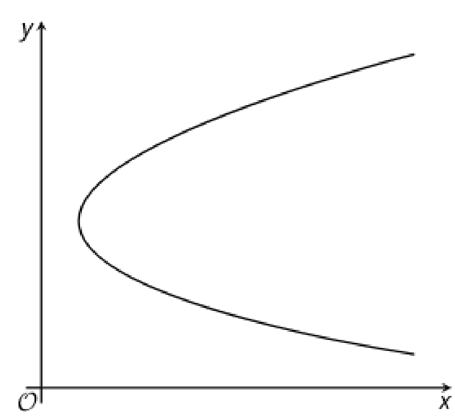
Years since published	**Number of copies sold**
0	
1	
2	
3	
y	

   b. Write an equation representing the number of copies, c, sold y years after the book was released.

   c. Use your equation to find c when $y = 6$. What does this mean in terms of the book?

(From Unit 6)

7. Explain why this graph does not represent a function.

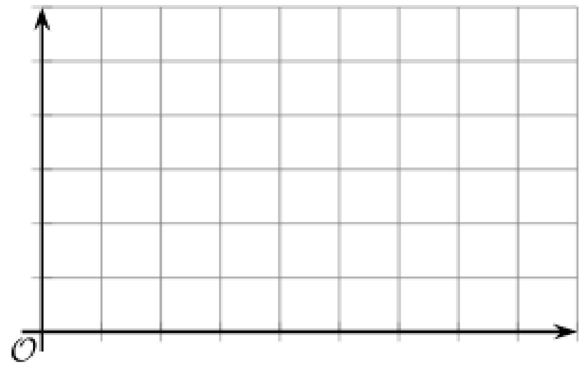


(From Unit 5)

8. Function D gives the height of a drone t seconds after it lifts off.

   Sketch a possible graph for this function given that:

   - $D(3) = 4$
   - $D(10) = 0$
   - $D(5) > D(3)$



(From Unit 5)

9. Solve this system of linear equations without graphing: $\begin{cases} 3y + 7 = 5x \\ 7x - 3y = 1 \end{cases}$

(From Unit 3)

10. Select **all** the expressions equivalent to $80 - 20x$.

   a. $20(4 - 20x)$

   b. $20(4 - x)$

   c. $-4(-20 + 5x)$

   d. $-10(8 - 2x)$

   e. $16(5 - 4x)$

(Addressing NC.7.EE.1)

Lesson 3: Comparing Quadratic and Exponential Functions

Learning Targets

- I can compare the rates of change for exponential functions to those for quadratic functions.

Bridge

Different students are evaluating two expressions, $3 \cdot 6^x$ and 5^x. Analyze their work, describe any errors made in the "corrected work" column, and evaluate each expression correctly.

	Noah's work	Mai's work	Corrected work
Evaluate 5^x when x is 6.	5^x 5^6 30	5^x 5^6 $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$ $7,776$	
Evaluate $3 \cdot 6^x$ when x is 2.	$3 \cdot 6^x$ $3 \cdot 6^2$ $3 \cdot 12$ 36	$3 \cdot 6^x$ $3 \cdot 6^2$ 18^2 324	

Warm-up: From Least to Greatest

List these quantities in order, from least to greatest, without evaluating each expression. Be prepared to explain your reasoning.

a. 2^{10} b. 10^2 c. 2^9 d. 9^2

Activity 1: Which One Grows Faster?

- In pattern A, the length and width of the rectangle grow by one small square from each figure to the next.
- In pattern B, the number of small squares doubles from each figure to the next.
- In each pattern, the number of small squares is a function of the figure number, n.

Pattern A

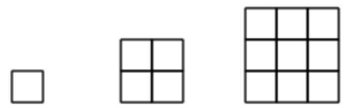


Figure 0 Figure 1 Figure 2 Figure 3

Pattern B

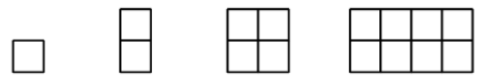


Figure 0 Figure 1 Figure 2 Figure 3

Pattern A

1. Write an equation to represent the number of small squares at figure n in pattern A.
2. Is the function linear, quadratic, or exponential?
3. Complete the table:

n, figure number	**$f(n)$, number of small squares**
0	
1	
2	
3	
4	
5	
6	
7	
8	

Pattern B

1. Write an equation to represent the number of small squares at figure n in pattern B.
2. Is the function linear, quadratic, or exponential?
3. Complete the table:

n, figure number	**$g(n)$, number of small squares**
0	
1	
2	
3	
4	
5	
6	
7	
8	

4. How would the two patterns compare if they continue to grow? Make one or two observations.

Activity 2: Comparing Two More Functions

Here are two functions: $p(x) = 6x^2$ and $q(x) = 3^x$.

Investigate the output of p and q for different values of x. As the value of x increases, which function will eventually have a greater value?

Support your answer with tables, graphs, or other representations.

Are You Ready For More?

1. Jada says that some exponential functions grow more slowly than the quadratic function as x increases. Do you agree with Jada? Explain your reasoning.

2. Could you have an exponential function $g(x) = b^x$ and a quadratic function $f(x) = x^2$ so that $g(x) < f(x)$ for all values of x?

Lesson Debrief

Lesson 3 Summary and Glossary

We have seen that the graphs of quadratic functions can curve upward. Graphs of exponential functions, with base larger than 1, also curve upward. To compare the two, let's look at the quadratic expression $3n^2$ and the exponential expression 2^n.

A table of values shows that $3n^2$ is initially greater than 2^n, but 2^n eventually becomes greater.

n	$3n^2$	2^n
1	3	2
2	12	4
3	27	8
4	48	16
5	75	32
6	108	64
7	147	128
8	192	256

We also saw an explanation for why exponential growth eventually overtakes quadratic growth.

- When n increases by 1, the exponential expression 2^n always increases by a factor of 2.
- The quadratic expression $3n^2$ increases by different factors, depending on n, but these factors get smaller. For example, when n increases from 2 to 3, the factor is $\frac{27}{12}$ or 2.25. When n increases from 6 to 7, the factor is $\frac{147}{108}$ or about 1.36. As n increases to larger and larger values, $3n^2$ grows by a factor that gets closer and closer to 1.

A quantity that always doubles will eventually overtake a quantity growing by this smaller factor at each step.

Unit 7 Lesson 3 Practice Problems

1. The table shows values of the expressions $10x^2$ and 2^x.

x	$10x^2$	2^x
1	10	2
2	40	4
3	90	8
4	160	16
8		
10		
12		

   a. Describe how the values of each expression change as x increases.

   b. Predict which expression will have a greater value when:

      i. x is 8

      ii. x is 10

      iii. x is 12

   c. Find the value of each expression when x is 8, 10, and 12.

   d. Make an observation about how the values of the two expressions change as x becomes greater and greater.

2. Function f is defined by $f(x) = 1.5^x$. Function g is defined by $g(x) = 500x^2 + 345x$.

   a. Which function is quadratic? Which one is exponential?

   b. The values of which function will eventually be greater for larger and larger values of x?

3. Create a table of values to show that the exponential expression $3(2)^x$ eventually overtakes the quadratic expression $3x^2 + 2x$.

x	$3(2)^x$	$3x^2 + 2x$

4. The table shows the values of 4^x and $100x^2$ for some values of x.

   Use the patterns in the table to explain why eventually the values of the exponential expression 4^x will overtake the values of the quadratic expression $100x^2$.

x	4^x	$100x^2$
1	4	100
2	16	400
3	64	900
4	256	1600
5	1024	2500

5. Here are some lengths, widths, and areas of a garden whose perimeter is 60 feet.

   a. What lengths and widths do you think will produce the largest possible area? Explain how you know.

   b. Complete the table with the missing measurements.

Length (ft)	Width (ft)	Area (sq feet)
3	27	81
6	24	
9		
12		
15		225
18		
21		189

(From Unit 7, Lesson 1)

6. A bicycle costs \$240, and it loses $\frac{3}{5}$ of its value each year.

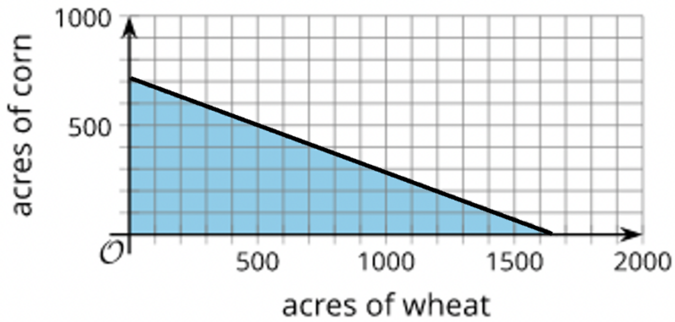
   a. Write expressions for the value of the bicycle, in dollars, after 1, 2, and 3 years.

   b. When will the bike be worth less than \$1?

   c. Will the value of the bike ever be 0? Explain your reasoning.

(From Unit 6)

7. A farmer plants wheat and corn. It costs about \$150 per acre to plant wheat and about \$350 per acre to plant corn. The farmer plans to spend no more than \$250,000 planting wheat and corn. The total area of corn and wheat that the farmer plans to plant is less than 1200 acres.



This graph represents the inequality $150w + 350c \le 250,000$, which describes the cost constraint in this situation. Let w represent the number of acres of wheat and c represent the number of acres of corn.

a. The inequality $w + c < 1,200$ represents the total area constraint in this situation. On the same coordinate plane, graph the solution to this inequality.

b. Use the graphs to find at least two possible combinations of the number of acres of wheat and the number of acres of corn that the farmer could plant.

c. The combination of 400 acres of wheat and 700 acres of corn meets one constraint in the situation but not the other constraint. Which constraint does this meet? Explain your reasoning.

(From Unit 3)

8. Jada is researching the price of vegetables at different grocery stores around her city. The mean prices per pound and standard deviation of the prices are shown in the table.

Store	Mean price per pound	Standard deviation
Veggies "R" Us	\$3.56	\$1.43
Veggiemart	\$3.56	\$0.89

What conclusions can Jada draw from these data?

(From Unit 4)

9. Solve the equation for x : $6x + 8y = 9$.

(From Unit 2)

10. Evaluate each of the following expressions if $x = 3$.

a. $4x^2$

b. $(4x)^2$

c. $2 \cdot 4^x$

d. $4 \cdot 2^x$

(Addressing NC.6.EE.1; NC.6.EE.2)

Lesson 4: Building Quadratic Functions to Describe Situations (Part One)

Learning Targets

- I can explain the meaning of each term in a quadratic expression that represents the height of a free-falling object.
- I can use tables, graphs and equations to represent the height of a free-falling object.
- I can write quadratic functions to represent the height of an object falling due to gravity.

Bridge

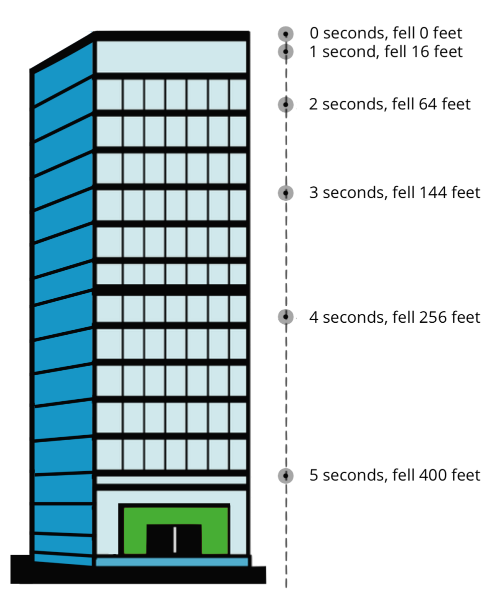
In the United States, weather temperatures are usually expressed using the Fahrenheit temperature scale. In many other countries, the weather temperatures are expressed using the Celsius temperature scale. The function $C(x) = \frac{5}{9}(x - 32)$ describes the relationship between the temperatures, where C represents the temperature in degrees Celsius and x represents the temperature in degrees Fahrenheit. If the high temperature in Charlotte averages 92° Fahrenheit in August, what is the average temperature in degrees Celsius?

Warm-up: An Interesting Numerical Pattern

Study the table. What do you notice? What do you wonder?

x	0	1	2	3	4	5
y	0	16	64	144	256	400

Activity 1: Falling from the Sky



A rock is dropped from the top floor of a 500-foot tall building. A camera captures the distance the rock traveled, in feet, after each second.

1. How far will the rock have fallen after 6 seconds? Show your reasoning.

2. Jada noticed that the distances fallen are all multiples of 16. She wrote down:

$$\begin{aligned} 16 &= 16 \cdot 1 \\ 64 &= 16 \cdot 4 \\ 144 &= 16 \cdot 9 \\ 256 &= 16 \cdot 16 \\ 400 &= 16 \cdot 25 \end{aligned}$$

Then, she noticed that 1, 4, 9, 16, and 25 are 1^2, 2^2, 3^2, 4^2 and 5^2.

a. Use Jada's observations to predict the distance fallen from an even taller building after 7 seconds. (Assume the building is tall enough that an object dropped from the top of it will continue falling for at least 7 seconds.) Show your reasoning.

b. Write an equation for the function, with d representing the distance dropped after t seconds.

Activity 2: Galileo and Gravity

Galileo Galilei, an Italian scientist, and other medieval scholars studied the motion of free-falling objects. The law they discovered can be expressed by the equation $d(t) = 16 \cdot t^2$, which gives the distance fallen in feet, d, as a function of time, t, in seconds.

An object is dropped from a height of 576 feet.

1. Evaluate $d(0.5)$ and explain what it means.

2. To find out where the object is after the first few seconds after it was dropped, Elena and Diego created different tables.

Elena's table:

Time (seconds)	Distance fallen (feet)
0	0
1	16
2	64
3	
4	
t	

Diego's table:

Time (seconds)	Distance from the ground (feet)
0	576
1	560
2	512
3	
4	
t	

a. How are the two tables alike? How are they different?

b. Complete Elena's and Diego's tables. Be prepared to explain your reasoning.

Lesson Debrief

Lesson 4 Summary and Glossary

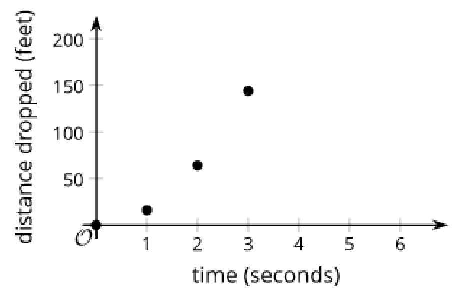
The distance traveled by a falling object in a given amount of time is an example of a quadratic function. Galileo is said to have dropped balls of different mass from the Leaning Tower of Pisa, which is about 190 feet tall, to show that they travel the same distance in the same time. In fact the equation $d(t) = 16t^2$ models the distance d, in feet, that a cannonball falls after t seconds, no matter what its mass.

Because $16 \cdot 4^2 = 256$, and the tower is only 190 feet tall, the cannonball hits the ground before 4 seconds.

Here is a table showing how far the cannonball has fallen over the first few seconds.

Time (seconds)	Distance fallen (feet)
0	0
1	16
2	64
3	144

Here are the time and distance pairs plotted on a coordinate plane:



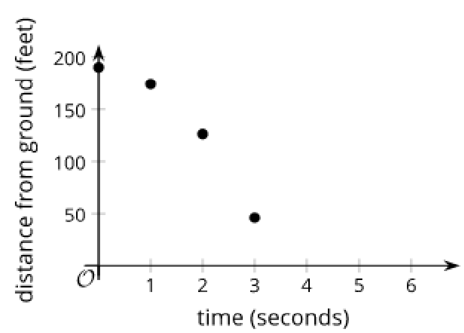
Notice that the distance fallen is increasing each second. The average rate of change is increasing each second, which means that the cannonball is speeding up over time. This comes from the influence of gravity, which is represented by the quadratic expression $16t^2$. It is the exponent 2 in that expression that makes it increase by larger and larger amounts.

Another way to study the change in the position of the cannonball is to look at its distance from the ground as a function of time.

Here is a table showing the distance from the ground in feet at 0, 1, 2, and 3 seconds.

Time (seconds)	Distance from the ground (feet)
0	190
1	174
2	126
3	46

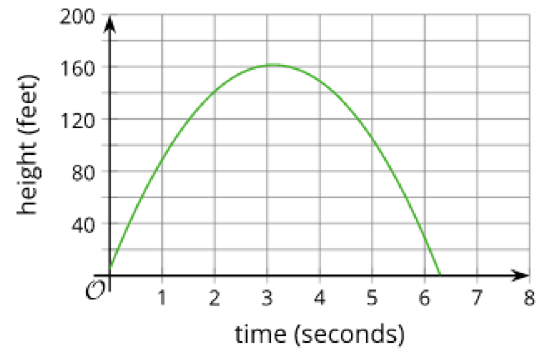
Here are the time and distance pairs plotted on a graph:



The function rule that defines the distance from the ground as a function of time is $d(t) = 190 - 16t^2$. It tells us that the cannonball's distance from the ground is 190 feet before it is dropped and has decreased by $16t^2$ when t seconds have passed.

Unit 7 Lesson 4 Practice Problems

1. A rocket is launched in the air and its height, in feet, is modeled by the function h. Here is a graph representing h.



   Select **all** true statements about the situation.

   a. The rocket is launched from a height less than 20 feet above the ground.

   b. The rocket is launched from about 20 feet above the ground.

   c. The rocket reaches its maximum height after about 3 seconds.

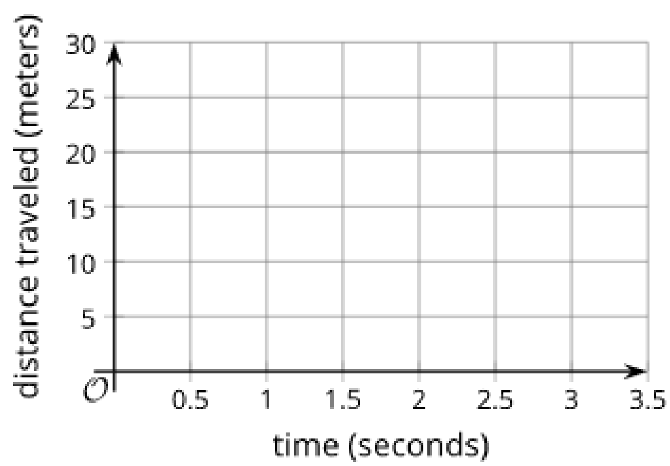
   d. The rocket reaches its maximum height after about 160 seconds.

   e. The maximum height of the rocket is about 160 feet.

2. A baseball travels d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

   a. Complete the table and plot the data on the coordinate plane.

t **(seconds)**	d **(meters)**



   b. Is the baseball traveling at a constant speed? Explain how you know.

3. A rock is dropped from a bridge over a river. Which table could represent the distance in feet fallen as a function of time in seconds?

Table A

Time (seconds)	**Distance fallen (feet)**
0	0
1	48
2	96
3	144

Table B

Time (seconds)	**Distance fallen (feet)**
0	0
1	16
2	64
3	144

Table C

Time (seconds)	**Distance fallen (feet)**
0	180
1	132
2	84
3	36

Table D

Time (seconds)	**Distance fallen (feet)**
0	180
1	164
2	116
3	36

4. A small ball is dropped from a tall building. Which equation could represent the ball's height, h, in feet, relative to the ground, as a function of time, t, in seconds?

   a. $h = 100 - 16t$

   b. $h = 100 - 16t^2$

   c. $h = 100 - 16^t$

   d. $h = 100 - \frac{16}{t}$

5. Determine whether $5n^2$ or 3^n will have the greater value when:

   a. $n = 1$

   b. $n = 3$

   c. $n = 5$

(From Unit 7, Lesson 3)

6. Diego claimed that $10 + x^2$ is always greater than 2^x and used this table as evidence. Do you agree with Diego?

x	$10 + x^2$	2^x
1	11	2
2	14	4
3	19	8
4	26	16

(From Unit 7, Lesson 3)

7. The table shows the height, in centimeters, of the water in a swimming pool at different times since the pool started to be filled.

Minutes	Height
0	150
1	150.5
2	151
3	151.5

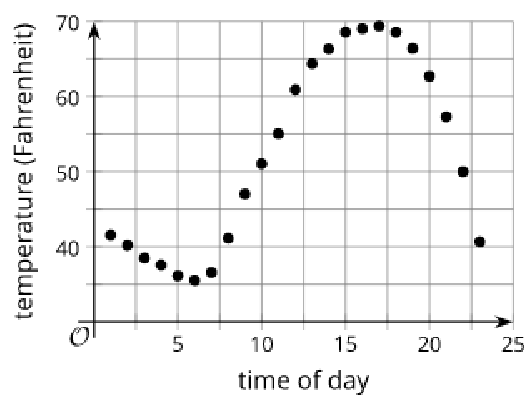
   a. Does the height of the water increase by the same amount each minute? Explain how you know.

   b. Does the height of the water increase by the same factor each minute? Explain how you know.

(From Unit 6)

8. The temperature was recorded several times during the day. Function T gives the temperature in degree Fahrenheit, n hours since midnight.

   Here is a graph for this function.



   a. Pick two consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

   b. Pick two non-consecutive points and connect them with a line segment. Estimate the slope of that line. Explain what that estimated value means in this situation.

(From Unit 5)

9. (*Technology required.*) A study investigated the relationship between the amount of daily food waste measured in pounds and the number of people in a household. The table displays the results of the study.

Number of people in household, x	Food waste (pounds), y
2	3.4
3	2.5
4	8.9
4	4.7
4	3.5
4	4
5	5.3
5	4.6
5	7.8
6	3.2
8	12

Use graphing technology to create the line of best fit for the data in the table.

a. What is the equation of the line of best fit for the data? Round numbers to two decimal places.

b. What is the slope of the line of best fit? What does it mean in this situation? Is this realistic?

c. What is the y-intercept of the line of best fit? What does it mean in this situation? Is this realistic?

(From Unit 4)

10. The function $F(c) = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius, gives temperature in degrees Fahrenheit based on the temperature in degrees Celsius. If the high temperature in Reykjavik, Iceland, on Christmas is $5°$ Celsius, what is the temperature in degrees Fahrenheit?

(Addressing NC.6.EE.2)

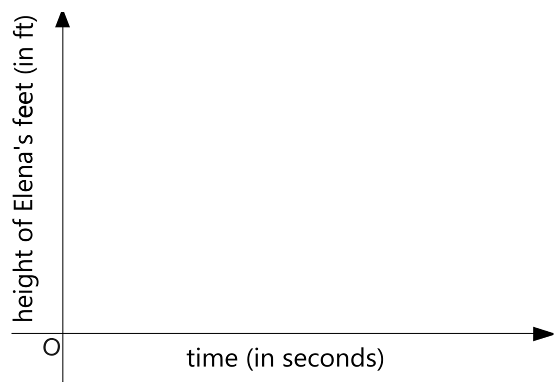
Lesson 5: Building Quadratic Functions to Describe Situations (Part Two)

Learning Targets

- I can create quadratic functions and graphs that represent a situation.
- I can relate the vertex of a graph and the zeros of a function to a situation.
- I know that the domain of a function can depend on the situation it represents.

Bridge

Elena jumps three times on a trampoline, with each jump going higher than the last! If the trampoline is 4 feet off the ground, draw a graph that could represent the height of Elena's feet after x seconds.



Warm-up: Sky Bound

A cannon is 10 feet off the ground. It launches a cannonball straight up with a velocity of 406 feet per second.

Imagine that there is no gravity and that the cannonball continues to travel upward with the same velocity.

1. Complete the table with the heights of the cannonball at different times.

Seconds	0	1	2	3	4	5	t
Distance above ground (feet)	10						

2. Write an equation to model the distance in feet, d, of the ball t seconds after it was fired from the cannon if there was no gravity.

Activity 1: Tracking a Cannonball

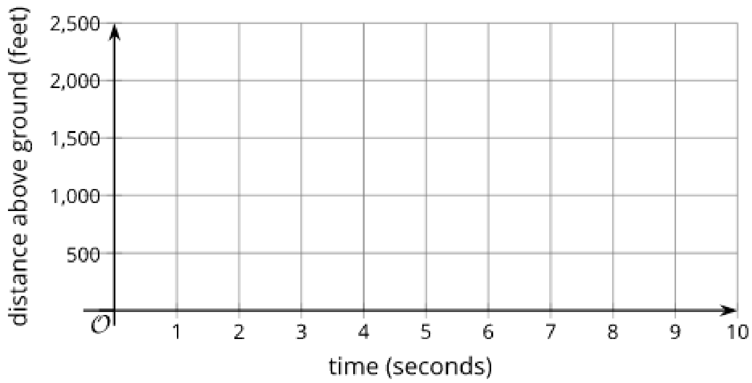
Earlier, you completed a table that represents the height of a cannonball, in feet, as a function of time, in seconds, if there was no gravity.

1. This table shows the actual heights of the ball at different times.

Seconds	0	1	2	3	4	5
Distance above ground (feet)	10	400	758	1,084	1,378	1,640

Compare the values in this table with those in the table you completed earlier. Make at least two observations.

a. Plot the two sets of data you have on the same coordinate plane.



b. How are the two graphs alike? How are they different?

2. Write an equation to model the actual distance d, in feet, of the ball t seconds after it was fired from the cannon. If you get stuck, consider the differences in distances and the effects of gravity from a previous lesson.

Activity 2: Graphing Another Cannonball

The function defined by $d = 50 + 312t - 16t^2$ gives the height in feet of a cannonball t seconds after the ball leaves the cannon.

1. What do the terms 50, $312t$, and $-16t^2$ tell us about the cannonball?

2. Use graphing technology to graph the function. Adjust the graphing window to the following boundaries: $0 \leq x \leq 25$ and $0 \leq y \leq 2{,}000$.

3. Observe the graph and:
    a. Describe the shape of the graph. What does it tell us about the movement of the cannonball?

    b. Estimate the maximum height the ball reaches. When does this happen?

    c. Estimate when the ball hits the ground.

4. What domain is appropriate for this function? Explain your reasoning.

5. What range is appropriate for this function? Explain your reasoning.

Are You Ready For More?

If the cannonball were fired at 800 feet per second, would it reach a mile in height? Explain your reasoning.

Lesson Debrief

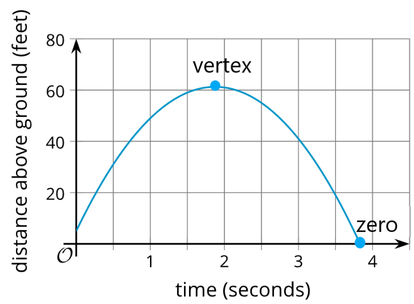
Lesson 5 Summary and Glossary

In this lesson, we looked at the height of objects that are launched upward and then come back down because of gravity.

An object is thrown upward from a height of 5 feet with a velocity of 60 feet per second. Its height, $h(t)$, in feet, after t seconds is modeled by the function $h(t) = 5 + 60t - 16t^2$.

- The linear expression $5 + 60t$ represents the height the object would have at time t if there were no gravity. The object would keep going up at the same speed at which it was thrown. The graph would be a line with a slope of 60, which comes from the constant speed of 60 feet per second.
- The expression $-16t^2$ represents the effect of gravity, which eventually causes the object to slow down, stop, and start falling back again.

Here is the graph of h:



The graph representing any quadratic function is a special kind of "U" shape called a **parabola**. You will learn more about the geometry of parabolas in a future course.

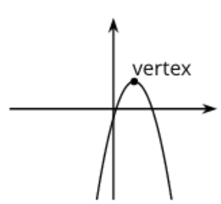
Notice the parabola intersects the vertical axis at 5, which means the object was thrown into the air from 5 feet off the ground. The graph indicates that the object reaches its peak height of about 60 feet after a little less than 2 seconds. That peak is the point on the graph where the function reaches a maximum value. At that point, the curve changes direction, and the output of the function changes from increasing to decreasing. We call that point the **vertex** of the graph. Every parabola has a vertex, because there is a point where it changes direction—from increasing to decreasing, or the other way around.

The object hits the ground a little before 4 seconds. That time corresponds to the horizontal intercept of the graph. An input value that produces an output of 0 is called a **zero** of the function. A zero of the function h is approximately 3.8 because $h(3.8) \approx 0$.

In this situation, input values less than 0 seconds or more than about 3.8 seconds would not be meaningful, so an appropriate domain for this function would include all values of t between 0 and about 3.8.

Parabola: The "U"-shaped graph representing any quadratic function.

Vertex : The vertex of the graph of a quadratic function is the point where the graph changes from increasing to decreasing or vice versa. It is the highest or lowest point on the graph.



Zero (of a function): An input that yields an output of zero. If other words, if $f(a) = 0$, then a is a zero of f.

Unit 7 Lesson 5 Practice Problems

1. The height of a diver above the water is given by $h(t) = -5t^2 + 10t + 3$, where t is time measured in seconds and $h(t)$ is measured in meters.

   Select **all** statements that are true about the situation.

   a. The diver begins 5 meters above the water.

   b. The diver begins 3 meters above the water.

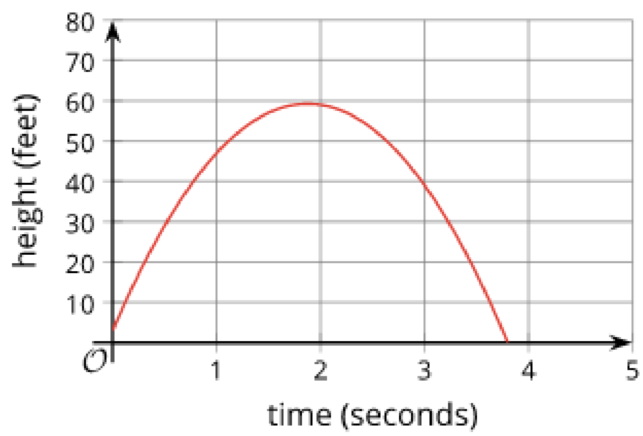
   c. The function has 1 zero that makes sense in this situation.

   d. The function has 2 zeros that make sense in this situation.

   e. The graph that represents h starts at the origin and curves upward.

   f. The diver begins at the same height as the water level.

2. The height of a baseball, in feet, is modeled by the function h given by the equation $h(t) = 3 + 60t - 16t^2$. The graph of the function is shown.



   a. About when does the baseball reach its maximum height?

   b. About how high is the maximum height of the baseball?

   c. About when does the ball hit the ground?

3. (*Technology required.*) Two rocks are launched straight up in the air. The height of rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is time measured in seconds, and height is measured in feet.

   Use graphing technology to graph both equations. Determine which rock hits the ground first and explain how you know.

4. Each expression represents an object's distance from the ground in meters as a function of time, t, in seconds.

   Object A: $-5t^2 + 25t + 50$

   Object B: $-5t^2 + 50t + 25$

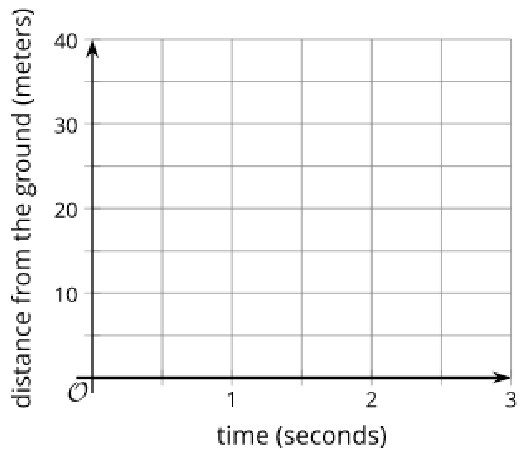
   a. Which object was launched with the greatest vertical speed?

   b. Which object was launched from the greatest height?

5. Han accidentally drops his water bottle from the balcony of his apartment building. The equation $d = 32 - 5t^2$ gives the distance from the ground, d, in meters, after t seconds.

   a. Complete the table and plot the data on the coordinate plane.

t **(seconds)**	d **(meters)**
0	
0.5	
1	
1.5	
2	



   b. Is the water bottle falling at a constant speed? Explain how you know.

(From Unit 7, Lesson 4)

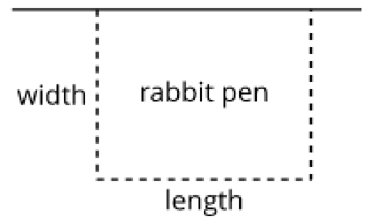
6. The function f is defined by $f(x) = 2^x$, and the function g is defined by $g(x) = x^2 + 16$.

   a. Find the values of f and g when x is 4, 5, and 6.

   b. Will the values of f always be greater than the values of g? Explain how you know.

(From Unit 7, Lesson 3)

7. Tyler is building a pen for their rabbit on the side of the garage. They need to fence in three sides of the pen and want to use 24 ft of fencing.



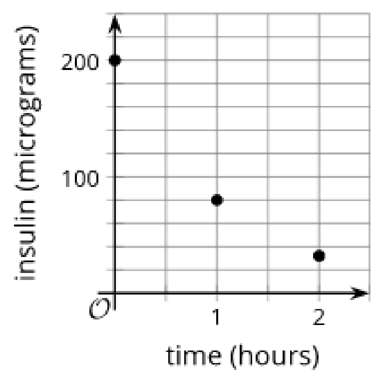
   a. The table shows some possible lengths and widths. Complete each area.

Length (ft)	Width (ft)	Area (sq ft)
8	8	
10	7	
12	6	
14	5	
16	4	

   b. Which length-and-width combination should Tyler choose to give their rabbit the most room?

(From Unit 7, Lesson 1)

8. The graph shows how much insulin, in micrograms (mcg), is in a patient's body after receiving an injection.



   a. Write an equation giving the number of mcg of insulin, m, in the patient's body h hours after receiving the injection.

   b. After 3 hours, will the patient still have at least 10 mcg of insulin in their body? Explain how you know.

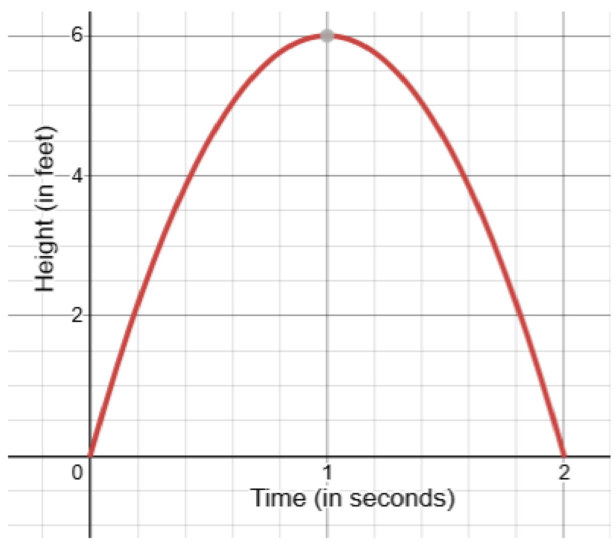
(From Unit 6)

9. Lin says that a solution to the equation $2x - 6 = 7x$ must also be a solution to the equation $5x - 6 = 10x$.

   Write a convincing explanation about why this is true.

   (From Unit 3)

10. The graph plots the path of a high jumper at Parabola High School competing in the state championships. What do you know about the jump from the graph?



   (Addressing NC.8.F.5)

Lesson 6: Building Quadratic Functions to Describe Situations (Part Three)

Learning Targets

- I can choose a domain that makes sense in a revenue situation.
- I can relate the vertex of a graph and the zeros of a function to revenue and profit situations.
- I can model revenue with quadratic functions and graphs.

Bridge

Mrs. Gillis gives her class a set of 36 math challenges to complete during the first quarter. Andre decides to complete four each week. The function $y = 36 - 4x$ represents the number of challenges Andre has remaining to complete, y, after x weeks.

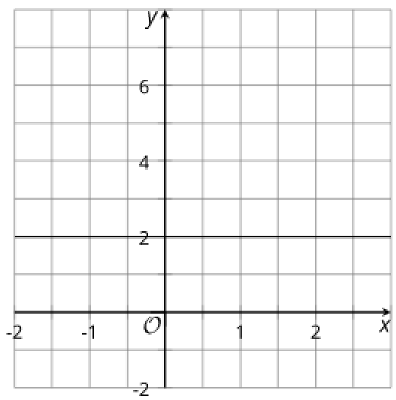
1. What is the y-intercept of this function? What does it represent?

2. What is the x-intercept of this function? What does it represent?

Warm-up: Graphs of Four Functions

Which one doesn't belong? Explain your reasoning.

a.



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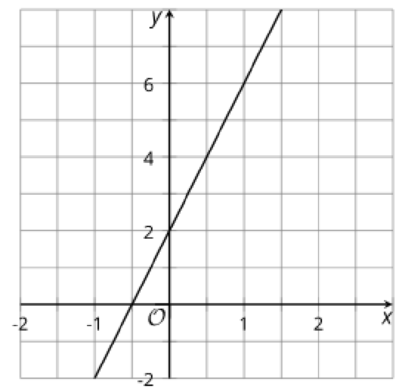
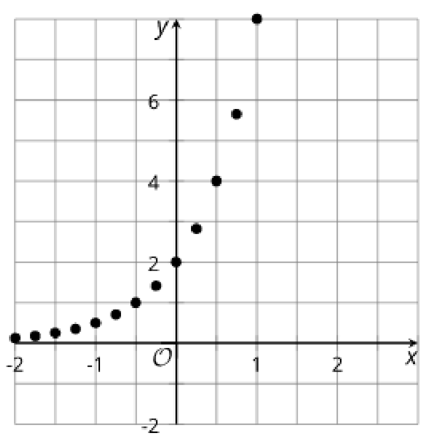
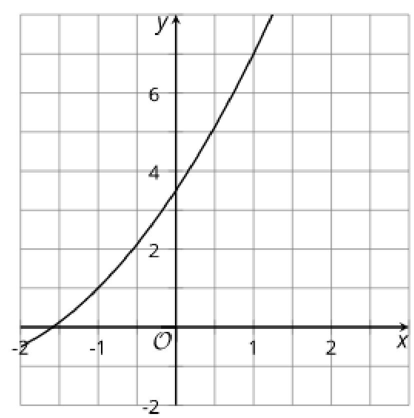


c.



d.



Activity 1: Pharmaceutical Profiting


(Image source[1])

The profit of a pharmaceutical company's insulin is modeled by the equation $p(v) = -0.3v^2 + 150v$, where p represents profit (in millions of dollars), and v represents the number of vials (in millions) of insulin sold.

1. Graph the function $p(v)$ in Desmos.

2. Identify an interval where the function is positive ($p(v) > 0$). Interpret the meaning of this interval in the context of the situation the function describes.

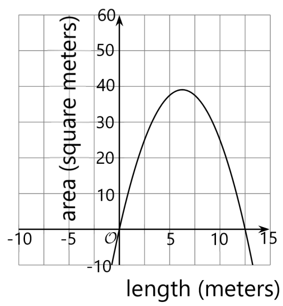
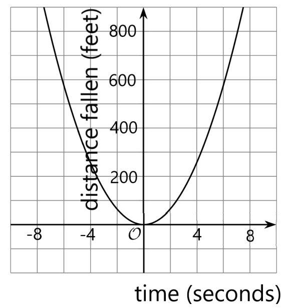
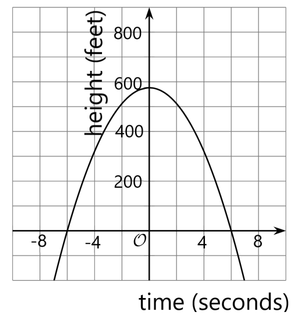
3. Identify an interval where the function is negative ($p(v) < 0$). Interpret the meaning of this interval in the context of the situation the function describes.

4. If the company doesn't make much insulin, people are willing to pay more for that insulin because it is so hard to find. If the company makes a lot of insulin, there is more insulin than people really need, so the company cannot charge as much. With your partner, discuss why the profit would be small when the company makes too much or too little insulin.

[1] Barbhuiya, T. (2021). Free injection. *Unsplash.* https://unsplash.com/photos/msQB97gUxY0

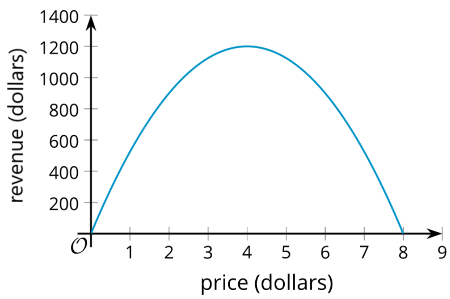
Activity 2: Domain, Range, Vertex, and Zeros

Here are three sets of descriptions and equations that represent some familiar quadratic functions. Graphs related to each function are also shown, though they may not reflect a reasonable domain for each function. For each function, complete the table.

	1. The area of rectangle with a perimeter of 25 meters and a side length x: $A(x) = x \cdot \frac{(25-2x)}{2}$ 	2. The distance in feet that an object has fallen t seconds after being dropped: $g(t) = 16t^2$ 	3. The height in feet of an object t seconds after being dropped: $h(t) = 576 - 16t^2$ 
Describe a domain that is appropriate for the situation. Think about any upper or lower limits for the input, as well as whether all numbers make sense as the input.			
Describe a range that is appropriate for the situation.			
Describe how the graph should be modified to show the domain (and range) that makes sense.			
Identify or estimate the vertex on the graph. Describe what it means in the situation.			
Identify or estimate the zeros of the function. Describe what it means in the situation.			

Lesson Debrief

Lesson 6 Summary and Glossary



As you have seen, quadratic functions often come up when studying revenue. (Recall that “revenue” means the money collected when someone sells something.) Interpreting the key features of these functions can help businesses make decisions about how much to charge.

Suppose we are selling raffle tickets and deciding how much to charge for each ticket. When the price of the tickets is higher, typically fewer tickets will be sold.

Let’s say that with a price of d dollars, it is possible to sell $600-75d$ tickets. A function that models the revenue r collected is $r(d)=d(600-75d)$, or $r(d)=600d-75d^2$. Here is a graph that represents the function.

If the greatest revenue is $1,200, and the revenue collected cannot be negative, then the range of the function r is between 0 and 1200.

We can also see that the domain of the function r is between 0 and 8. Clearly the cost of the tickets cannot be negative. If the cost of the tickets were more than $8, the expression for the number of tickets sold, $600-75d$, becomes negative. Since the number of tickets sold cannot be negative, this tells us our model does not work for $d>0$.

The graph shows that the zeros of the function r are 0 and 8. These are the prices for which we would make no money from the raffle.

As the raffle organizers, we are most interested in setting the price of tickets to make the most money. We can see this in the vertex of the graph, (4, 1200). If we charge $4 for the tickets, we will produce the maximum revenue of $1200.

Unit 7 Lesson 6 Practice Problems

1. Based on past musical productions, a theater predicts selling $400 - 8p$ tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many tickets the theater expects to sell and what revenues it expects to receive at the given ticket prices.

Ticket price (dollars)	Number of tickets sold	Revenue (dollars)
5		
10		
15		
20		
30		
45		
50		
p		

   b. For which ticket prices will the theater earn no revenue? Explain how you know.

   c. At what price should the theater sell the tickets if it must earn at least $3,200 in revenue to break even (to not lose money) on the musical production? Explain how you know.

2. A company sells running shoes. If the price of a pair of shoes in dollars is p, the company estimates that it will sell $50,000 - 400p$ pairs of shoes.

   Write an expression that represents the revenue in dollars from selling running shoes if a pair of shoes is priced at p dollars.

3. (*Technology required.*) The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If s is the sale price of the item, then profit, $P(s)$, can be represented by the function rule: $P(s) = -2s^2 + 24s - 54$.

   a. Graph the function $P(s)$ in Desmos.

   b. Identify an interval where the function is positive ($P(s) > 0$). Interpret the meaning of this interval in the context of the situation the function describes.

   c. Identify an interval where the function is negative ($P(s) < 0$). Interpret the meaning of this interval in the context of the situation the function describes.[2]

[2] Adapted from https://tasks.illustrativemathematics.org/

5. The function f represents the revenue in dollars the school can expect to receive if it sells $220 - 12x$ coffee mugs for x dollars each.

   Here is the graph of f.

   Select **all** the statements that describe this situation.

   a. At \$2 per coffee mug, the revenue will be \$196.

   b. The school expects to sell 160 mugs if the price is \$5 each.

   c. The school will lose money if it sells the mugs for more than \$10 each.

   d. The school will earn about \$1,000 if it sells the mugs for \$10 each.

   e. The revenue will be more than \$700 if the price is between \$4 and \$14.

   f. The expected revenue will increase if the price per mug is greater than \$10.

6. (*Technology required.*) A small marshmallow is launched straight up in the air with a slingshot. The function h, given by the equation $h(t) = 5 + 20t - 5t^2$, describes the height of the marshmallow in meters as a function of time, t, in seconds, since it was launched.

   a. Use Desmos to graph the function h.

   b. About when does the marshmallow reach its maximum height?

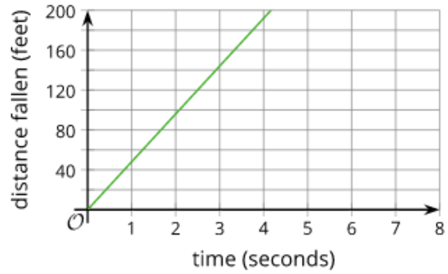
   c. About how long does it take before the marshmallow hits the ground?

   d. What domain makes sense for the function h in this situation?

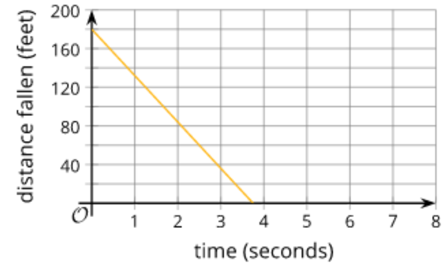
(From Unit 7, Lesson 4)

7. A rock is dropped from a bridge over a river. Which graph could represent the distance fallen, in feet, as a function of time in seconds? Explain your reasoning.

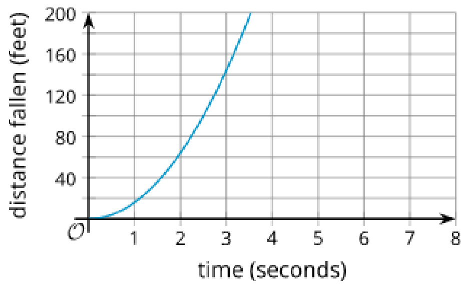
Graph A



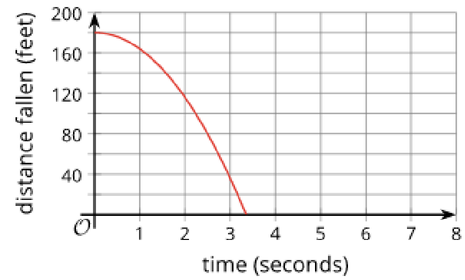
Graph B



Graph C



Graph D



(From Unit 7, Lesson 4)

8. A bacteria population, p, is modeled by the equation $p = 100,000 \cdot 2^d$, where d is the number of days since the population was first measured.

   Assuming the population growth has been the same prior to the day it was first measured, select **all** the predictions that are true statements in this situation.

   a. $100,000 \cdot 2^{-2}$ represents the bacteria population 2 days before it was first measured.

   b. The bacteria population 3 days before it was first measured was 800,000.

   c. The population was more than 1,000 one week before it was first measured.

   d. The population was more than 1,000,000 one week after it was first measured.

   e. The bacteria population 4 days before it was first measured was 6,250.

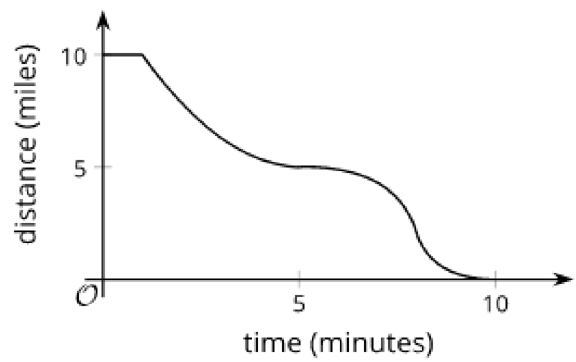
(From Unit 6)

9. Simplify the expression, writing with all positive exponents: $\frac{18x^5y^{-2}z^3}{12x^6y^4z^{-2}}$

(From Unit 6)

10. Here is a graph of Han's distance from home as he drives.

Identify the intercepts of the graph and explain what they mean in terms of Han's distance from home.



(From Unit 5)

11. Throughout the month of January, the weather in Charlotte gets colder. According to timeanddate.com, on January 1, 2021, the high temperature was $54°$ Fahrenheit, and it dropped about $0.5°$F per day for 30 days, until January 31, 2021. Mai models the temperature in Charlotte by function $t = -0.5d + 54$, where t represents the temperature and d represents the number of days passed.

    a. What is the vertical intercept of this function? What does it represent?

    b. What is the horizontal intercept of this function? What does it represent?

    c. Is the horizontal intercept of this function realistic? Why or why not?

    (Addressing NC.8.F.4)

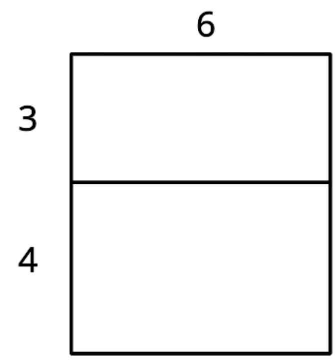
Lesson 7: Equivalent Quadratic Expressions

Learning Targets

- I can rewrite quadratic expressions in different forms by using an area diagram or the distributive property.

Warm-up: Diagrams of Products

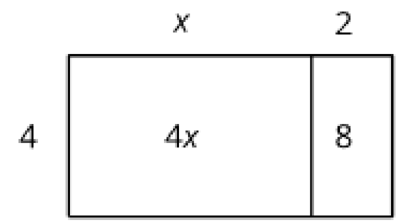
1. Explain why the diagram shows that $6(3+4)=6\cdot 3+6\cdot 4$.



2. Draw a diagram to show that $5(x+2)=5x+10$.

Activity 1: Drawing Diagrams to Represent More Products

Applying the distributive property or expanding $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.



1. Draw a diagram to show that $n(2n+5)$ and $2n^2+5n$ are equivalent expressions.

2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

   a. $6(\frac{1}{3}n+2)$

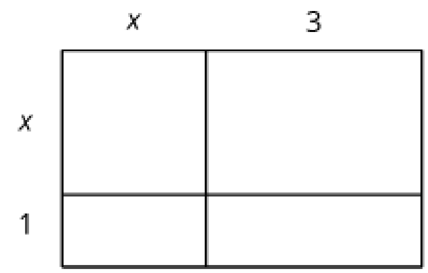
   b. $p(4p+9)$

   c. $5r(r+\frac{3}{5})$

   d. $(0.5w+7)w$

Activity 2: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x+1$ and $x+3$. Use this diagram to show that $(x+1)(x+3)$ and x^2+4x+3 are equivalent expressions.



2. Draw diagrams to help you write an equivalent expression for each of the following:

   a. $(x+5)^2$

   b. $2x(x+4)$

   c. $(2x+1)(x+3)$

   d. $(x+m)(x+n)$

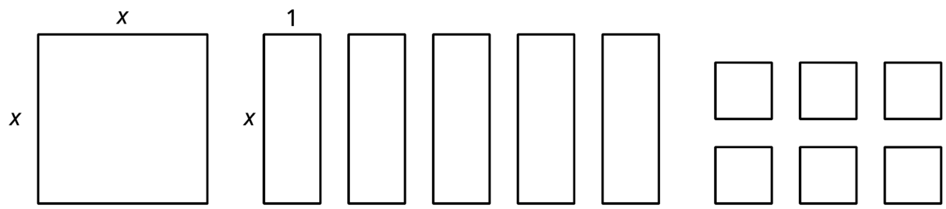
3. Write an equivalent expression for each expression without drawing a diagram:

   a. $(x+2)(x+6)$

   b. $(x+5)(2x+10)$

Are You Ready For More?

1. Is it possible to arrange an x-by-x square, five x-by-1 rectangles, and six 1-by-1 squares into a single large rectangle? Explain or show your reasoning.



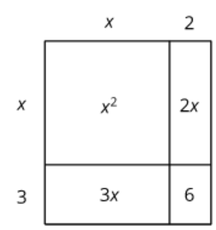
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?

3. Is there a different non-zero number of 1-by-1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

Lesson Debrief

Lesson 7 Summary and Glossary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed as $x(18-x)$, which can also be written as $18x-x^2$. The former is a product of x and $18-x$, and the latter is a difference of $18x$ and x^2, but both expressions represent the same function.



Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x+2)(x+3)$. We can write an equivalent expression by thinking about each factor, the $(x+2)$ and $(x+3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.

Multiplying $(x+2)$ and $(x+3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x+2)(x+3)$ is equivalent to $x^2+2x+3x+6$, or to x^2+5x+6.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x+2$) is multiplied by every term in the other factor (the x and the 3 in $x+3$).

$$
\begin{aligned}
&(x+2)(x+3)\\
&= x(x+3)+2(x+3)\\
&= x^2+3x+2x+(2)(3)\\
&= x^2+(3+2)x+(2)(3)
\end{aligned}
$$

In general, when a quadratic expression is written in the form of $(x+p)(x+q)$, we can apply the distributive property to rewrite it as $x^2+px+qx+pq$ or $x^2+(p+q)x+pq$.

Unit 7 Lesson 7 Practice Problems

1. Draw a diagram to show that $(2x+5)(x+3)$ is equivalent to $2x^2+11x+15$.

2. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

   a. $(x+2)(x+6)$
   b. $(2x+8)(x+2)$
   c. $(x+8)(x+4)$
   d. $(x+2)(2x+6)$

   1. $x^2+12x+32$
   2. $2x^2+10x+12$
   3. $2x^2+12x+16$
   4. $x^2+8x+12$

3. Select **all** expressions that are equivalent to x^2+4x.

   a. $x(x+4)$
   b. $(x+2)^2$
   c. $(x+x)(x+4)$
   d. $(x+2)^2-4$
   e. $(x+4)x$

4. Tyler drew a diagram to expand $(x+5)(2x+3)$.

   a. Explain Tyler's mistake.

	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

   b. What is the correct expanded form of $(x+5)(2x+3)$?

5. Based on past concerts, a band predicts selling $600-10p$ concert tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many concert tickets the band expects to sell and what revenues it expects to receive at the given ticket prices.

Ticket price (dollars)	**Number of tickets**	**Revenue (dollars)**
10		
15		
20		
30		
35		
45		
50		
60		
p		

   b. In this model, at what ticket prices will the band earn no revenue at all?

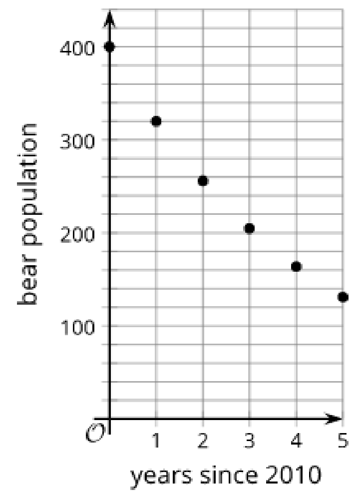
   c. At what price should the band sell the tickets if it must earn at least 8,000 dollars in revenue to break even (to not lose money) on a given concert. Explain how you know.

(From Unit 7, Lesson 6)

6. Explain why the values of the exponential expression 3^x will eventually overtake the values of the quadratic expression $10x^2$.

(From Unit 7, Lesson 3)

7. A population of bears decreases exponentially.



   a. What is the annual decay factor for the bear population? Explain how you know.

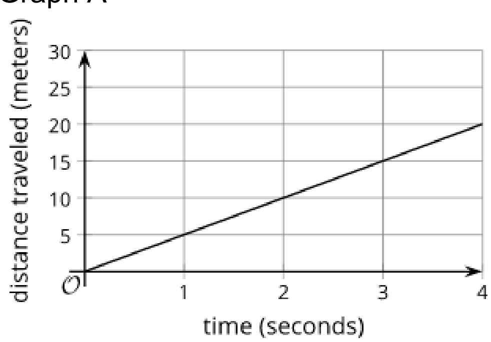
   b. Using function notation, represent the relationship between the bear population, b, and the number of years since the population was first measured, t. That is, find a function f so that $b = f(t)$.

(From Unit 6)

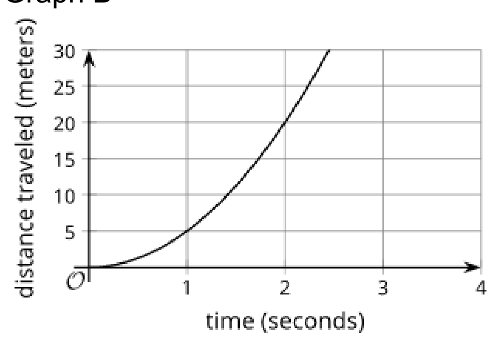
8. A baseball travels d meters in t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

Which graph could represent this situation? Explain how you know.

Graph A



Graph B



(From Unit 6)

9. Equations defining functions a, b, c, d, and f are shown here.

   Select **all** the equations that represent exponential functions.

   a. $a(x) = 2^3 \cdot x$

   b. $b(t) = (\frac{2}{3})^t$

   c. $c(m) = \frac{1}{5} \cdot 2^m$

   d. $d(x) = 3x^2$

   e. $f(t) = 3 \cdot 2^t$

(From Unit 6)

10. Consider a function q defined by $q(x) = x^2$. Explain why negative values are not included in the range of q.

(From Unit 5)

Lesson 8: Standard Form and Factored Form

Learning Targets

- I know the difference between “standard form” and “factored form.”
- I can rewrite quadratic expressions given in factored form in standard form using either the distributive property or a diagram.

Bridge

1. Here are two diagrams that could be used to multiply $47 \cdot 62$. Which diagram do you think is most helpful and why?[1]

Diagram A

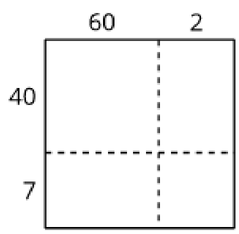
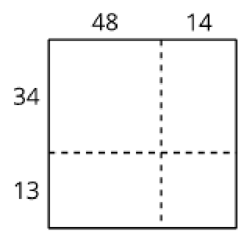
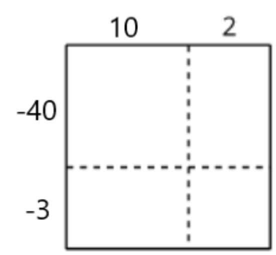


Diagram B



2. To multiply $-43 \cdot 12$, a student drew this diagram, found the product for each of the four boxes, and then found their sum. Does this diagram work with a negative factor? Explain.



3. Would a diagram work to multiply $-36 \cdot -19$? If so, draw the diagram.

Warm-up: Opposites Attract

Solve each equation mentally.

1. $40 - 8 = 40 + n$
2. $25 + -100 = 25 - n$
3. $3 - \frac{1}{2} = 3 + n$
4. $72 - n = 72 + 6$

Activity 1: Finding Products of Differences

1. Show that $(x-1)(x-1)$ and $x^2 - 2x + 1$ are equivalent expressions by drawing a diagram or applying the distributive property. Show your reasoning.

2. For each expression, write an equivalent expression. Show your reasoning.

   a. $(x+1)(x-1)$

   b. $(x-2)(x+3)$

   c. $(x-2)^2$

Activity 2: What Is the Standard Form? What Is the Factored Form?

The quadratic expression x^2+4x+3 is written in standard form.

Here are some other quadratic expressions. The expressions on the left are written in standard form, and the expressions on the right are not.

Written in standard form:	Not written in standard form:
x^2-1	$(2x+3)x$
x^2+9x	$(x+1)(x-1)$
$\frac{1}{2}x^2$	$3(x-2)^2+1$
$4x^2-2x+5$	$-4(x^2+x)+7$
$-3x^2-x+6$	$(x+8)(-x+5)$
$1-x^2$	

1. What are some characteristics of expressions in standard form?

2. $(x+1)(x-1)$, $(2x+3)x$, and $(x+8)(-x+5)$ in the right column are quadratic expressions written in **factored form**. Why do you think that form is called factored form?

Are You Ready For More?

Which quadratic expression can be described as being both standard form and factored form? Explain how you know.

Lesson Debrief

Lesson 8 Summary and Glossary

A quadratic function can often be represented by many equivalent expressions. For example, a quadratic function f might be defined by $f(x) = x^2 + 3x + 2$. The quadratic expression $x^2 + 3x + 2$ is said to be in **standard form**.

Standard form (of a quadratic expression)**:** The standard form of a quadratic expression is $ax^2 + bx + c$, where a, b, and c are constants, and a is not 0.

In standard form, we refer to a as the **coefficient** of the squared term x^2, b as the **coefficient** of the **linear term** x, and c as the **constant term**. In $f(x) = x^2 + 3x + 2$, $a = 1$, $b = 3$, and $c = 2$.

Coefficient: In an algebraic expression, the coefficient of a variable is the constant the variable is multiplied by. If the variable appears by itself then it is regarded as being multiplied by 1 and the coefficient is 1. The coefficient of x in the expression $3x + 2$ is 3. The coefficient of p in the expression $5 + p$ is 1.

Constant term: In an expression like $5x + 2$, the number 2 is called the constant term because it doesn't change when $x x$ changes. In the expression $5x - 8$, the constant term is -8, because we think of the expression as $5x + (-8)$. In the expression $12x - 4$, the constant term is -4.

Linear term: The linear term in a quadratic expression (in standard form) $ax^2 + bx + c$, where a, b, and c are constants, is the term bx. (If the expression is not in standard form, it may need to be rewritten in standard form first.)

The function f can also be defined by the equivalent expression $(x+2)(x+1)$. When the quadratic expression is a product of two factors where each one is a linear expression, this is called the **factored form**. The expression $3(x-7)(x+1)$ is also in factored form: the product of a number and two linear expressions.

Factored form (of a quadratic expression)**:** A quadratic expression that is written as the product of a constant times two linear factors is said to be in factored form. For example, $2(x-1)(x+3)$ and $(5x+2)(3x-1)$ are both in factored form.

An expression in factored form can be rewritten in standard form by expanding it, which means multiplying out the factors. In a previous lesson, we saw how to use a diagram and to apply the distributive property to multiply two linear expressions, such as $(x+3)(x+2)$. We can do the same to expand an expression with a sum and a difference, such as $(x+5)(x-2)$, or to expand an expression with two differences, such as $(x-4)(x-1)$.

To represent $(x-4)(x-1)$ with a diagram, we can think of subtraction as adding the opposite:

$$
\begin{aligned}
&(x-4)(x-1)\\
&=(x+-4)(x+-1)\\
&=x(x+-1)+-4(x+-1)\\
&=x^2+-1x+-4x+(-4)(-1)\\
&=x^2+-5x+4\\
&=x^2-5x+4
\end{aligned}
$$

	x	-4
x	x^2	$-4x$
-1	$-x$	4

Unit 7 Lesson 8 Practice Problems

1. Write each quadratic expression in standard form. Draw a diagram if needed.

   a. $(x+4)(x-1)$

   b. $(2x-1)(3x-1)$

2. Consider the expression $8-6x+x^2$.

   a. Is the expression in standard form? Explain how you know.

   b. Is the expression equivalent to $(x-4)(x-2)$? Explain how you know.

3. Which quadratic expression is written in standard form?

   a. $(x+3)x$

   b. $(x+4)^2$

   c. $-x^2-5x+7$

   d. $x^2+2(x+3)$

4. Explain why $3x^2$ can be said to be in both standard form and factored form.

5. (*Technology required.*) Two rocks are launched straight up in the air. In both functions, t is time measured in seconds, and height is measured in feet.

   - The height of rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$.
   - The height of rock B is given by function g, where $g(t) = 5 + 20t - 16t^2$.

   Use graphing technology to graph both equations.

   a. What is the maximum height of each rock?

   b. Which rock reaches its maximum height first? Explain how you know.

(From Unit 7, Lesson 5)

6. A football player throws a football. The function h given by $h(t) = 6 + 75t - 16t^2$ describes the football's height in feet t seconds after it is thrown.

   Select **all** the statements that are true about this situation.

   a. The football is thrown from ground level.

   b. The football is thrown from 6 feet off the ground.

   c. In the function h, $-16t^2$ represents the effect of gravity.

   d. The outputs of h decrease and then increase in value.

   e. The function h has 2 zeros that make sense in this situation.

   f. The vertex of the graph of h gives the maximum height of the football.

(From Unit 7, Lesson 5)

7. Jada dropped her sunglasses from a bridge over a river. Which equation could represent the distance, y, fallen in feet as a function of time, t, in seconds?

   a. $y = 16t^2$

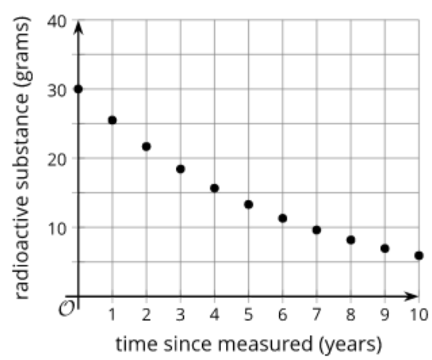
   b. $y = 48t$

   c. $y = 180 - 16t^2$

   d. $y = 180 - 48t$

(From Unit 7, Lesson 4)

8. The graph shows the number of grams of a radioactive substance in a sample at different times after the sample was first analyzed.



   a. What is the average rate of change for the substance during the 10-year period?

   b. Is the average rate of change a good measure for the change in the radioactive substance during these 10 years? Explain how you know.

(From Unit 6)

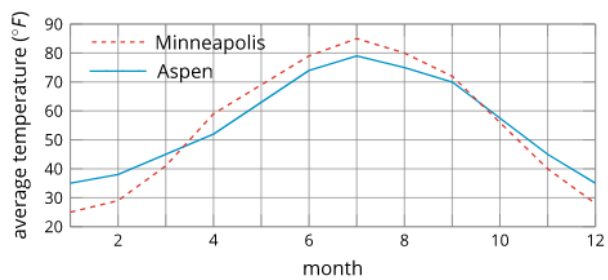
9. Each day after an outbreak of a new strain of the flu virus, a public health scientist receives a report of the number of new cases of the flu reported by area hospitals.

Days since outbreak	2	3	4	5	6	7
Number of new cases of the flu	20	28	38	54	75	105

Would a linear or exponential model be more appropriate for these data? Explain how you know.

(From Unit 6)

10. $A(t)$ is the average high temperature in Aspen, Colorado, t months after the start of the year. $M(t)$ is the temperature in Minneapolis, Minnesota, t months after the start of the year. Temperature is measured in degrees Fahrenheit.



a. What does $A(8)$ mean in this situation? Estimate $A(8)$.

b. Which city had a higher average temperature in February?

c. Were the two cities' average high temperatures ever the same? If so, when?

(From Unit 5)

11. Show two different strategies to multiply 23×54.

(Addressing NC.4.NBT.5)

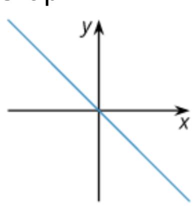
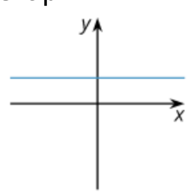
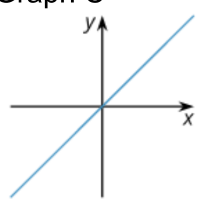
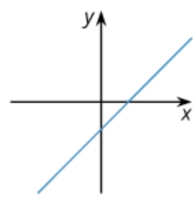
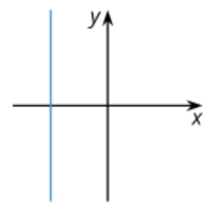
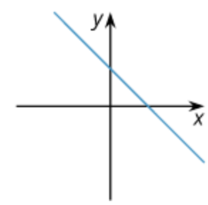
Lesson 9: Graphs of Functions in Standard and Factored Forms

Learning Targets

- I know how the numbers in the factored form of a quadratic expression relate to the intercepts of its graph.
- I can explain the meaning of the intercepts on a graph of a quadratic function in terms of the situation it represents.

Bridge

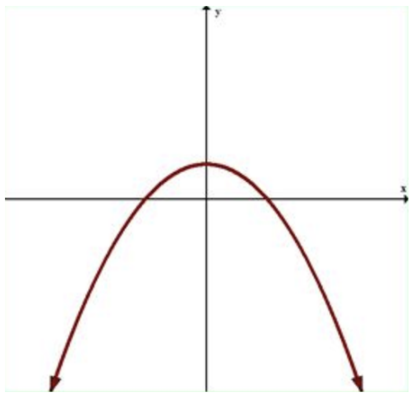
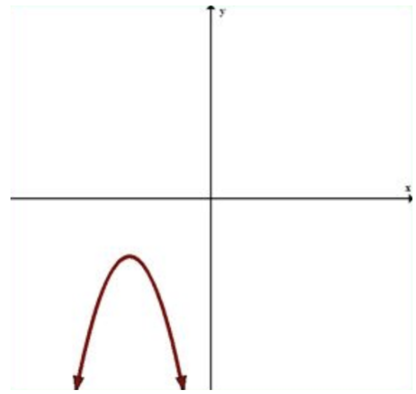
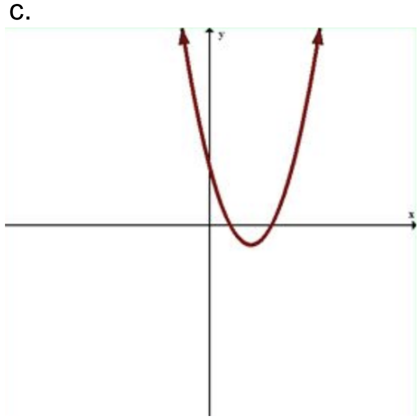
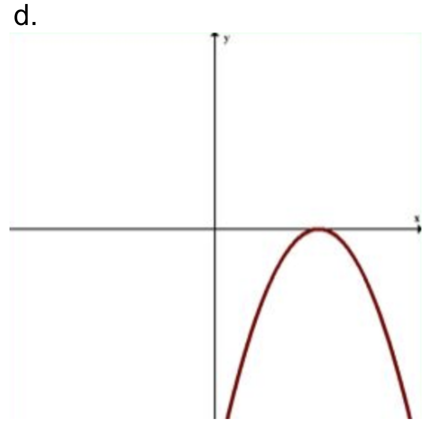
Here are some equations and graphs. Without technology, match each graph to one or more equations that it *could* represent. Be prepared to explain how you know.

Graph A	Graph B	Graph C	Graph D	Graph E	Graph F
					

- $y = 8$
- $y = 3x - 2$
- $x + y = 6$
- $0.5x = -4$
- $y = x$
- $-\frac{2}{3}x = y$
- $12 - 4x = y$
- $x - y = 12$
- $2x + 4y = 16$
- $3x = 5y$

Warm-up: Quadratic Quandary

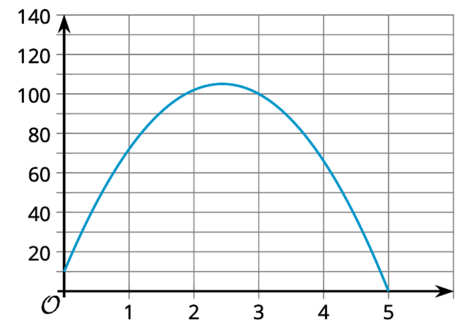
Which graph doesn't belong? Explain your reasoning.[1]

a. 	b. 
c. 	d. 

[1] Adapted with permission from www.wodb.ca.

Activity 1: Revisiting Projectile Motion

In an earlier lesson, we saw that an equation such as $h(t) = 10 + 78t - 16t^2$ can model the height of an object thrown upward from a height of 10 feet with a vertical velocity of 78 feet per second.



1. Is the expression $10 + 78t - 16t^2$ written in standard form? Explain how you know.

2. Jada said that the equation $g(t) = (-16t - 2)(t - 5)$ also defines the same function, written in factored form. Show that Jada is correct.

3. Here is a graph representing both $g(t) = (-16t - 2)(t - 5)$ and $h(t) = 10 + 78t - 16t^2$.
   a. Identify or approximate the vertical and horizontal intercepts.

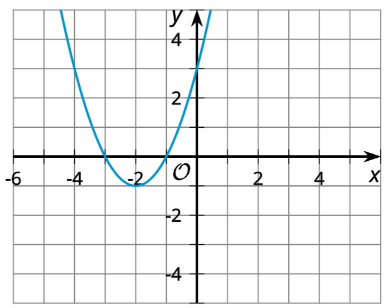
   b. What do each of these points mean in this situation?

Activity 2: Relating Expressions and Their Graphs

Here are pairs of expressions in standard and factored forms. Each pair of expressions define the same quadratic function, which can be represented with the given graph.

1. Identify the x-intercepts and the y-intercept of each graph.

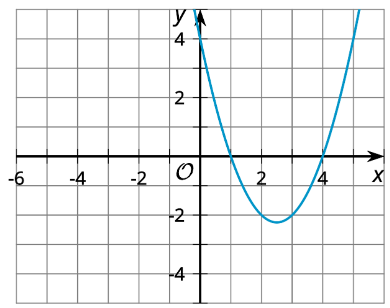
Function f
$x^2 + 4x + 3$
$(x + 3)(x + 1)$



x-intercepts: ______

y-intercept: ______

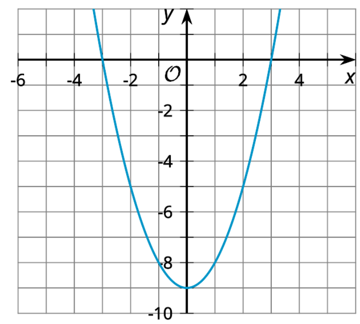
Function g
$x^2 - 5x + 4$
$(x - 4)(x - 1)$



x-intercepts: ______

y-intercept: ______

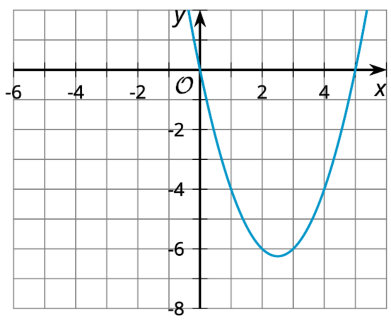
Function h
$x^2 - 9$
$(x - 3)(x + 3)$



x-intercepts: ______

y-intercept: ______

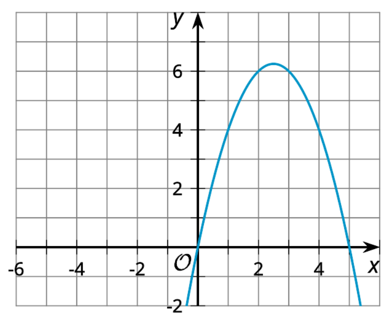
Function i
$x^2 - 5x$
$x(x - 5)$



x-intercepts: ______

y-intercept: ______

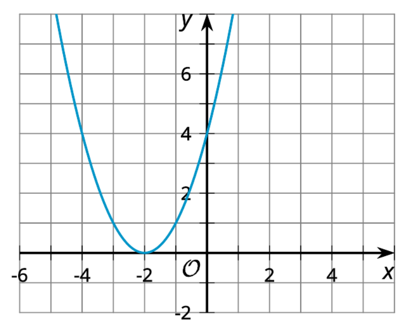
Function j
$5x - x^2$
$x(5 - x)$



x-intercepts: ______

y-intercept: ______

Function k
$x^2 + 4x + 4$
$(x + 2)(x + 2)$



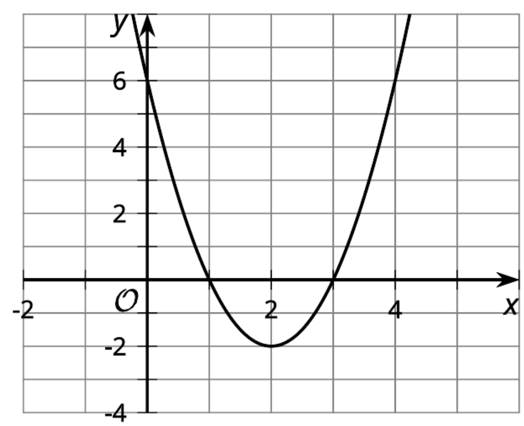
x-intercept: ______

2. What do you notice about the x-intercepts, the y-intercept, and the numbers in the expressions defining each function? Make a couple of observations.

3. Here is an expression that models function p, another quadratic function: $(x-9)(x-1)$. Predict the x-intercepts and the y-intercept of the graph that represent this function.

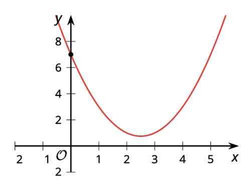
Are You Ready For More?

Find the values of a, p, and q that will make $y=a(x-p)(x-q)$ be the equation represented by the graph.

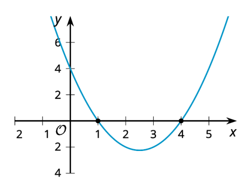


Lesson Debrief

Lesson 9 Summary and Glossary

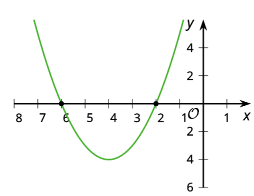


Different forms of quadratic functions can tell us interesting information about the function’s graph. When a quadratic function is expressed in standard form, it can tell us the y-intercept of the graph representing the function. For example, the graph representing $y = x^2 - 5x + 7$ has y-intercept $(0, 7)$. This makes sense because the y-coordinate is the y-value when x is 0. Evaluating the expression at $x = 0$ gives $y = 0^2 - 5(0) + 7$, which equals 7.



When a function is expressed in factored form, it can help us see the x-intercepts of its graph. Let’s look at the functions f given by $f(x) = (x-4)(x-1)$ and g given by $g(x) = (x+2)(x+6)$.

If we graph $y = f(x)$, we see that the x-intercepts of the graph are $(1, 0)$ and $(4, 0)$. Notice that “1” and “4” also appear in $f(x) = (x-4)(x-1)$, and they are subtracted from x.



If we graph $y = g(x)$, we see that the x-intercepts are at $(-2, 0)$ and $(-6, 0)$. Notice that “2” and “6” are also in the equation $g(x) = (x+2)(x+6)$, but they are added to x.

The connection between the factored form and the x-intercepts of the graph tells us about the zeros of the function (the input values that produce an output of 0). In the next lesson, we will further explore these connections between different forms of quadratic expressions and the graphs representing them.

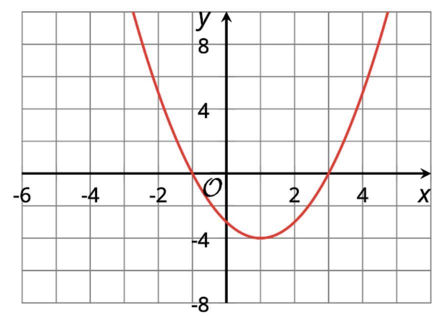
Unit 7 Lesson 9 Practice Problems

1. A quadratic function f is defined by $f(x) = (x-7)(x+3)$.

   a. Without graphing, identify the x-intercepts of the graph of f. Explain how you know.

   b. Expand $(x-7)(x+3)$ and use the expanded form to identify the y-intercept of the graph of f.

2. Here is a graph that represents a quadratic function. Which expression could define this function?



   a. $(x+3)(x+1)$

   b. $(x+3)(x-1)$

   c. $(x-3)(x+1)$

   d. $(x-3)(x-1)$

3.

   a. What is the y-intercept of the graph of the equation $y = x^2 - 5x + 4$?

   b. An equivalent way to write this equation is $y = (x-4)(x-1)$. What are the x-intercepts of this equation's graph?

4. Noah said that if we graph $y = (x-1)(x+6)$, the x-intercepts will be at $(1,0)$ and $(-6,0)$. Explain how you can determine, without graphing, whether Noah is correct.

5. Write each quadratic expression in standard form. Draw a diagram if needed.

   a. $(x-3)(x-6)$

   b. $(x-4)^2$

   c. $(2x+3)(x-4)$

   d. $(4x-1)(3x-7)$

(From Unit 7, Lesson 8)

6. A company sells a video game. If the price of the game in dollars is p, the company estimates that it will sell $20,000 - 500p$ games.

   Which expression represents the revenue in dollars from selling games if the game is priced at p dollars?

   a. $(20,000 - 500p) + p$

   b. $(20,000 - 500p) - p$

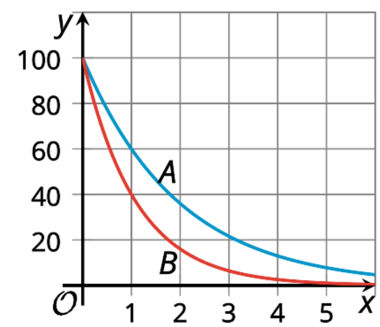
   c. $\frac{20,000-500p}{p}$

   d. $(20,000 - 500p) \cdot p$

(From Unit 7, Lesson 6)

7. Here are graphs of the functions f and g given by $f(x) = 100 \cdot (\frac{3}{5})^x$ and $g(x) = 100 \cdot (\frac{2}{5})^x$.

   Which graph corresponds to f and which graph corresponds to g? Explain how you know.



(From Unit 6)

8. Here are graphs of two functions f and g.

   An equation defining f is $f(x) = 100 \cdot 2^x$.

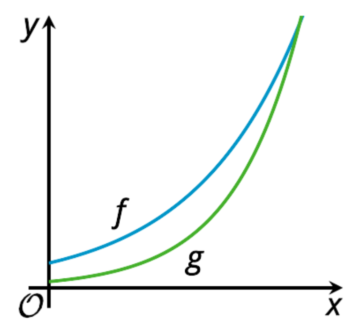
   Which of these could be an equation defining the function g?

   a. $g(x) = 25 \cdot 3^x$

   b. $g(x) = 50 \cdot (1.5)^x$

   c. $g(x) = 100 \cdot 3^x$

   d. $g(x) = 200 \cdot (1.5)^x$



(From Unit 6)

9. Elena plays the piano for 30 minutes each practice day. The total number of minutes, p, that Elena practiced last week is a function of n, the number of practice days.

   Find the domain and range for this function.

   (From Unit 5)

10. I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?[2]

    (Addressing NC.8.EE.7)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 10: Graphing from the Factored Form

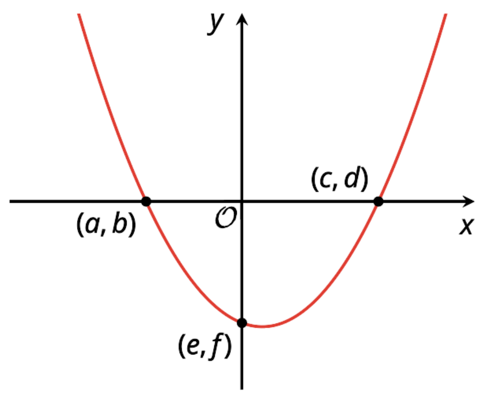
Learning Targets

- I can graph a quadratic function given in factored form.
- I know how to find the vertex and y-intercept of the graph of a quadratic function in factored form without graphing it first.

Warm-up: Finding Coordinates

Here is a graph of a function w defined by $w(x) = (x + 1.6)(x - 2)$. Three points on the graph are labeled.

Find the values of a, b, c, d, e, and f. Be prepared to explain your reasoning.



Activity 1: Comparing Two Graphs

Consider two functions defined by $f(x) = x(x+4)$ and $g(x) = x(x-4)$.

1. Complete the table of values for each function. Then, determine the x-intercepts and vertex of each graph. Be prepared to explain how you know.

x	$f(x)$
-5	5
-4	
-3	
-2	-4
-1	-3
0	
1	
2	
3	
4	32
5	

x-intercepts:

Vertex:

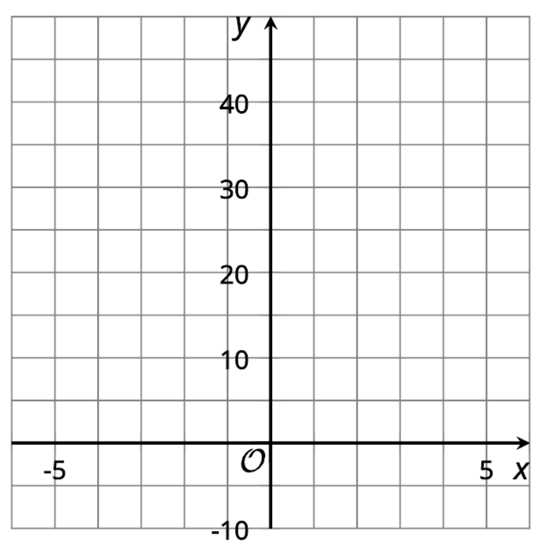
x	$g(x)$
-5	45
-4	
-3	
-2	12
-1	5
0	
1	
2	
3	-3
4	
5	

x-intercepts:

Vertex:

2. Plot the points from the tables on the same coordinate plane. (Consider using different colors or markings for each set of points so you can tell them apart.)

   Then, make a couple of observations about how the two graphs compare.



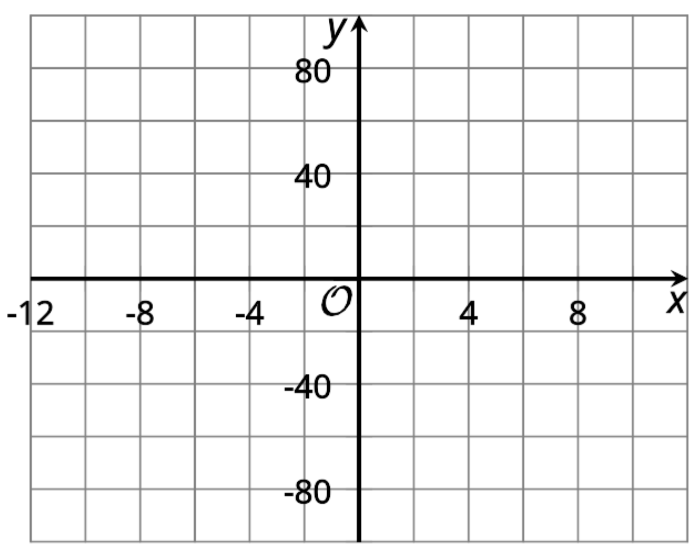
Activity 2: What Do We Need to Sketch a Graph?

1. The functions f, g, and h are given. Predict the x-intercepts and the x-coordinate of the vertex of each function.

Equation	x-intercepts	x-coordinate of the vertex
$f(x)=(x+3)(x-5)$		
$g(x)=2x(x-3)$		
$h(x)=(x+4)(4-x)$		

2. Use graphing technology to graph the functions f, g, and h. Use the graphs to check your predictions.

3. Without technology, sketch a graph that represents the equation $y=(x-7)(x+11)$ and that shows the x-intercepts and the vertex. Think about how to find the y-coordinate of the vertex. Be prepared to explain your reasoning.



Are You Ready For More?

The quadratic function f is given by $f(x) = x^2 + 2x + 6$.

1. Find $f(-2)$ and $f(0)$.

2. What is the x-coordinate of the vertex of the graph of this quadratic function?

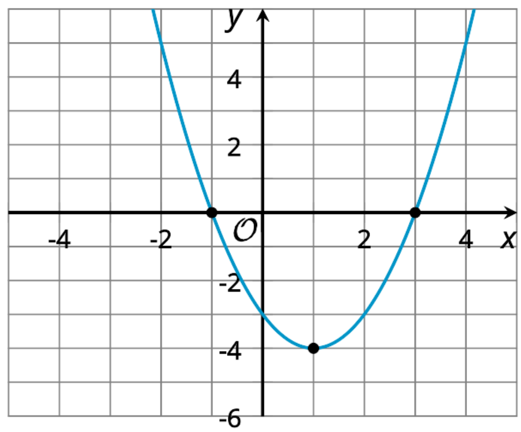
3. Does the graph have any x-intercepts? Explain or show how you know.

Lesson Debrief

Lesson 10 Summary and Glossary

The function f given by $f(x) = (x+1)(x-3)$ is written in factored form. Recall that this form is helpful for finding the zeros of the function (where the function has the value 0) and telling us the x-intercepts on the graph representing the function.

Here is a graph representing f. It shows 2 x-intercepts at $x = -1$ and $x = 3$.



If we use -1 and 3 as inputs to f, what are the outputs?

- $f(-1) = (-1+1)(-1-3) = (0)(-4) = 0$
- $f(3) = (3+1)(3-3) = (4)(0) = 0$

Because the inputs -1 and 3 produce an output of 0, they are the zeros of the function f. And because both x values have 0 for their y value, they also give us the x-intercepts of the graph (the points where the graph crosses the x-axis, which always have a y-coordinate of 0). So, the zeros of a function have the same values as the x-coordinates of the x-intercepts of the graph of the function.

The factored form can also help us identify the vertex of the graph, which is the point where the function reaches its minimum value. Notice that the x-coordinate of the vertex is 1, and that 1 is halfway between -1 and 3. Once we know the x-coordinate of the vertex, we can find the y-coordinate by evaluating the function at that x-coordinate: $f(1) = (1+1)(1-3) = 2(-2) = -4$. So the vertex is at $(1, -4)$.

When a quadratic function is in standard form, the y-intercept is clear: its y-coordinate is the constant term c in $ax^2 + bx + c$. To find the y-intercept from factored form, we can evaluate the function at $x = 0$, because the y-intercept is the point where the graph has an input value of 0: $f(0) = (0+1)(0-3) = (1)(-3) = -3$.

Unit 7 Lesson 10 Practice Problems

1. Select **all** true statements about the graph that represents $y = 2x(x - 11)$.
    a. Its x-intercepts are at $(-2, 0)$ and $(11, 0)$.
    b. Its x-intercepts are at $(0, 0)$ and $(11, 0)$.
    c. Its x-intercepts are at $(2, 0)$ and $(-11, 0)$.
    d. It has only one x-intercept.
    e. The x-coordinate of its vertex is -4.5.
    f. The x-coordinate of its vertex is 11.
    g. The x-coordinate of its vertex is 4.5.
    h. The x-coordinate of its vertex is 5.5.

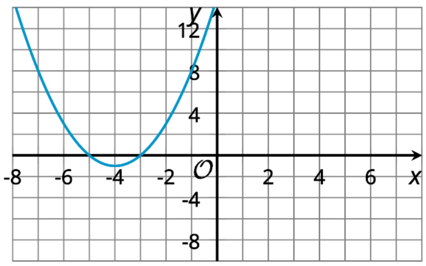
2. Select **all** equations whose graphs have a vertex with x-coordinate 2.
    a. $y = (x - 2)(x - 4)$
    b. $y = (x - 2)(x + 2)$
    c. $y = (x - 1)(x - 3)$
    d. $y = x(x + 4)$
    e. $y = x(x - 4)$

3. Determine the x-intercepts and the x-coordinate of the vertex of the graph that represents each equation.

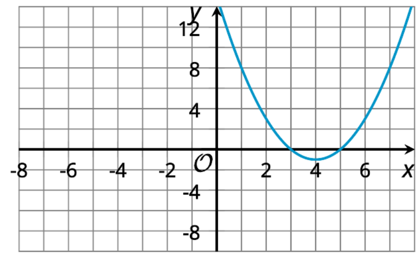
Equation	**x-intercepts**	**x-coordinates of the vertex**
$y = x(x - 2)$		
$y = (x - 4)(x + 5)$		
$y = -5x(3 - x)$		

4. Which graph below is the graph of the equation $y=(x-3)(x+5)$?

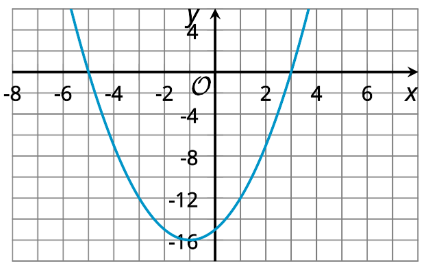
Graph A



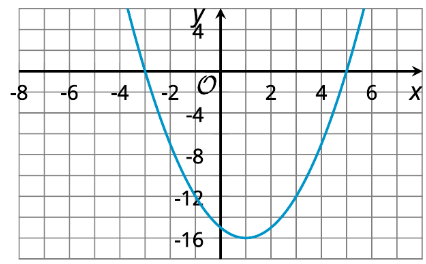
Graph B



Graph C



Graph D



5.

a. What are the x-intercepts of the graph of $y=(x-2)(x-4)$?

b. Find the coordinates of the vertex of the graph. Show your reasoning.

c. Sketch a graph of the equation $y=(x-2)(x-4)$.

6. What are the x-intercepts of the graph of the function defined by $(x-2)(2x+1)$?

   a. $(2,0)$ and $(-1,0)$

   b. $(2,0)$ and $(-\frac{1}{2},0)$

   c. $(-2,0)$ and $(1,0)$

   d. $(-2,0)$ and $(\frac{1}{2},0)$

7. Is $(s+t)^2$ equivalent to $s^2+2st+t^2$? Explain or show your reasoning.

   (From Unit 7, Lesson 7)

8. A company sells calculators. If the price of the calculator in dollars is p, the company estimates that it will sell $10,000-120p$ calculators.

   Write an expression that represents the revenue in dollars from selling calculators if a calculator is priced at p dollars.

   (From Unit 7, Lesson 6)

9. Which function could represent the height in meters of an object thrown upwards from a height of 25 meters above the ground t seconds after being launched?

   a. $f(t) = -5t^2$

   b. $f(t) = -5t^2 + 25$

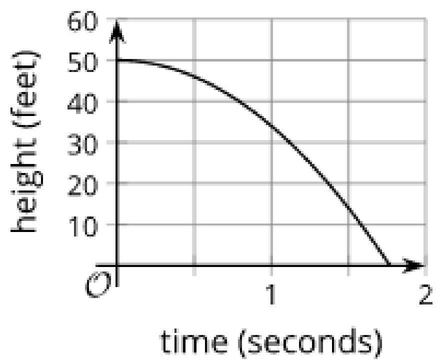
   c. $f(t) = -5t^2 + 25t + 50$

   d. $f(t) = -5t^2 + 50t + 25$

(From Unit 7, Lesson 5)

10. A basketball is dropped from the roof of a building, and its height in feet is modeled by the function h.

    Here is a graph representing h. Select **all** the true statements about this situation.



   a. When $t = 0$, the height is 0 feet.

   b. The basketball falls at a constant speed.

   c. The expression that defines h is linear.

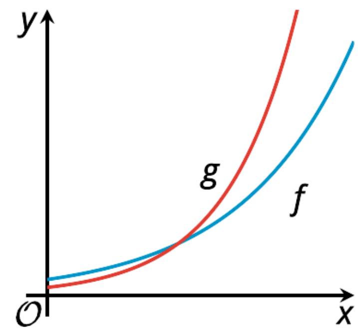
   d. The expression that defines h is quadratic.

   e. When $t = 0$, the ball is about 50 feet above the ground.

   f. The basketball lands on the ground about 1.75 seconds after it is dropped.

(From Unit 7, Lesson 4)

11. Here are graphs of two exponential functions f and g.



The function f is given by $f(x) = 100 \cdot 2^x$ while g is given by $g(x) = a \cdot b^x$.

Based on the graphs of the functions, what can you conclude about a and b?

(From Unit 6)

12. Suppose G takes a student's grade and gives a student's name as the output. Explain why G is not a function.

(From Unit 5)

Lesson 11: Graphing the Standard Form (Part One)

Learning Targets

- I can explain how the a and c in $y = ax^2 + bx + c$ affect the graph of the equation.
- I understand how graphs, tables, and equations that represent the same quadratic function are related.
- I can use technology to visualize the parameters of a quadratic expression in standard form.

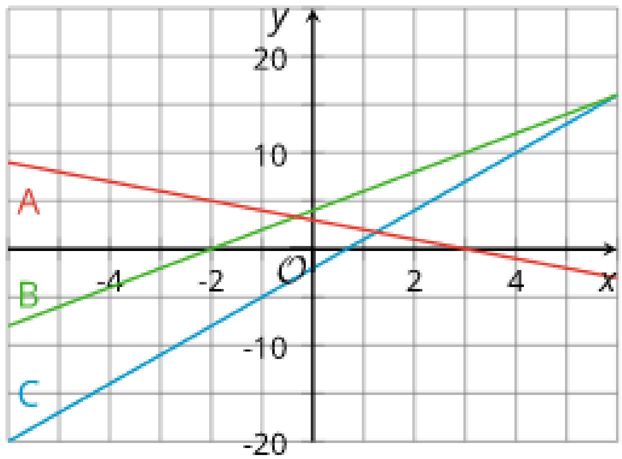
Bridge

For each of the following functions with the given inputs, predict whether the output will be positive or negative. Then, evaluate the functions. Were you correct? Why or why not?

Function	**Prediction**	**Evaluation**	**Correct prediction? Explain why or why not.**
a. $y = x^2$, where $x = 5$			
b. $y = x^2$, where $x = -5$			
c. $y = 2x^2$, where $x = 5$			
d. $y = 2x^2$, where $x = -5$			
e. $y = -2x^2$, where $x = 5$			
f. $y = -2x^2$, where $x = -5$			

Warm-up: Matching Graphs to Linear Equations

Graphs A, B, and C represent three linear equations: $y = 2x + 4$, $y = 3 - x$, and $y = 3x - 2$. Which graph corresponds to which equation? Explain your reasoning.



Equation	Graph	Explain your reasoning
$y = 2x + 4$		
$y = 3 - x$		
$y = 3x - 2$		

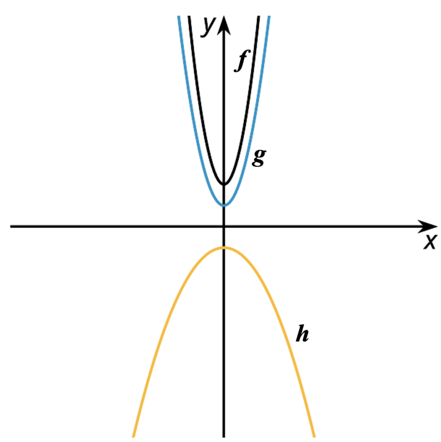
Activity 1: Quadratic Graphs Galore

Using graphing technology, graph $y = x^2$ and then experiment with each of the following changes to the function. Record your observations (include sketches, if helpful).

1. Adding different constant terms to x^2 (for example: $x^2 + 5$, $x^2 + 10$, $x^2 - 3$, etc.).
2. Multiplying x^2 by different positive coefficients greater than 1 (for example: $3x^2$, $7.5x^2$, etc.).
3. Multiplying x^2 by different negative coefficients less than or equal to -1 (for example: $-x^2$, $-4x^2$, etc.).
4. Multiplying x^2 by different coefficients between -1 and 1 (for example: $\frac{1}{2}x^2$, $-0.25x^2$, etc.).

Are You Ready For More?

Here are the graphs of three quadratic functions. What can you say about the coefficients of x^2 in the expressions that define f (in black at the top center), g (in blue on the top outside), and h (in yellow at the bottom)? Can you identify them? How do they compare?



Activity 2: What Do These Tables Reveal?

1.

a. Complete the table with values of $x^2 + 10$ and $x^2 - 3$ at different values of x.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x^2 + 10$							
$x^2 - 3$							

b. Earlier, you observed the effects on the graph of adding or subtracting a constant term from x^2. Study the values in the table. Use them to explain why the graphs changed the way they did when a constant term is added or subtracted.

2.

a. Complete the table with values of $2x^2$, $\frac{1}{2}x^2$, and $-2x^2$ at different values of x. (You may also use a spreadsheet tool, if available.)

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$							
$\frac{1}{2}x^2$							
$-2x^2$							

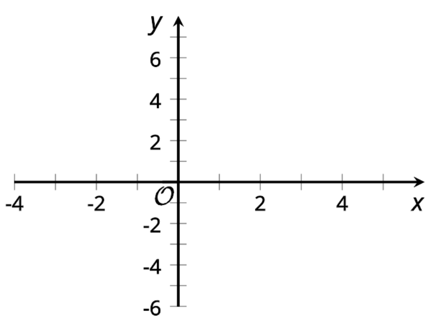
b. You also observed the effects on the graph of multiplying x^2 by different coefficients. Study the values in the table. Use them to explain why the graphs changed they way they did when x^2 is multiplied by a number greater than 1, by a negative number less than or equal to -1, and by numbers between -1 and 1.

Activity 3: Card Sort: Representations of Quadratic Functions

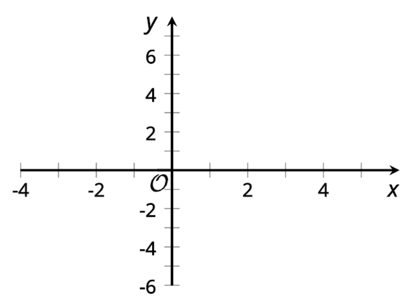
Your teacher will give your group a set of cards. Each card contains a graph or an equation.

- Take turns with your partner to sort the cards into sets so that each set contains two equations and a graph that all represent the same quadratic function.
- For each set of cards that you put together, explain to your partner how you know they belong together.
- For each set that your partner puts together, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
- Once all the cards are sorted and discussed, record the equivalent equations, sketch the corresponding graph, and write a brief note or explanation about why the representations were grouped together.

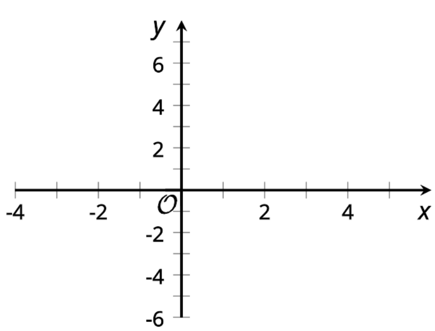
<table>
<tr>
<td>Standard form:

Factored form:


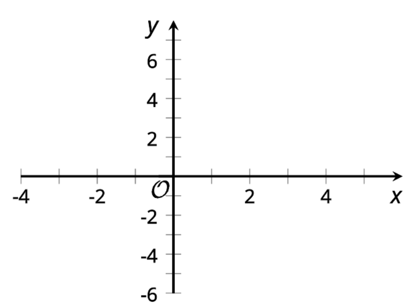
Explanation:</td>
<td>Standard form:

Factored form:


Explanation:</td>
</tr>
<tr>
<td>Standard form:

Factored form:


Explanation:</td>
<td>Standard form:

Factored form:


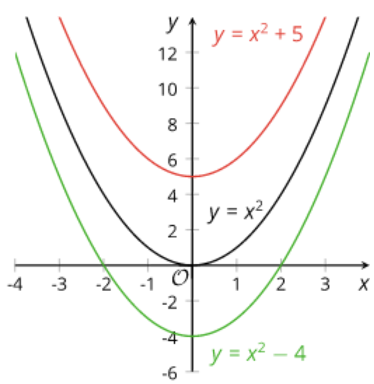
Explanation:</td>
</tr>
</table>

Lesson Debrief

Lesson 11 Summary and Glossary

Remember that the graph representing any quadratic function is a shape called a *parabola*. People often say that a parabola "opens upward" when the lowest point on the graph is the vertex (where the graph changes direction), and "opens downward" when the highest point on the graph is the vertex. Each coefficient in a quadratic expression written in standard form $ax^2 + bx + c$ tells us something important about the graph that represents it.

The graph of $y = x^2$ is a parabola opening upward with vertex at $(0, 0)$. Adding a constant term 5 gives $y = x^2 + 5$ and raises the graph by 5 units. Subtracting 4 from x^2 gives $y = x^2 - 4$ and moves the graph 4 units down.



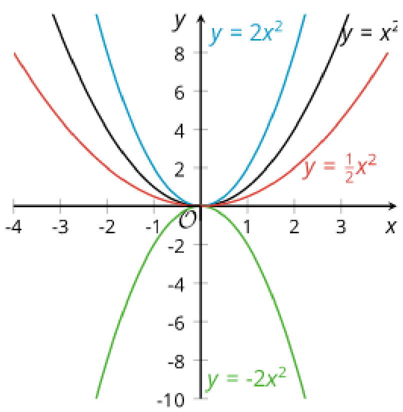
x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x^2 + 5$	14	9	6	5	6	9	14
$x^2 - 4$	5	0	-3	-4	-3	0	5

A table of values can help us see that adding 5 to x^2 increases all the output values of $y = x^2$ by 5, which explains why the graph moves up 5 units. Subtracting 4 from x^2 decreases all the output values of $y = x^2$ by 4, which explains why the graph shifts down by 4 units.

In general, the constant term of a quadratic expression in standard form influences the vertical position of the graph. An expression with no constant term (such as x^2 or $x^2 + 9x$) means that the constant term is 0, so the y-intercept of the graph is on the x-axis. It's not shifted up or down relative to the x-axis.

The coefficient of the squared term in a quadratic function also tells us something about its graph. The coefficient of the squared term in $y = x^2$ is 1. Its graph is a parabola that opens upward.

- Multiplying x^2 by a number greater than 1 makes the graph steeper, so the parabola is narrower than that representing x^2.
- Multiplying x^2 by a number less than 1 but greater than 0 makes the graph less steep, so the parabola is wider than that representing x^2.
- Multiplying x^2 by a number less than 0 makes the parabola open downward.



x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$	18	8	2	0	2	8	18
$-2x^2$	-18	-4	-2	0	-2	-8	-18

If we compare the output values of $2x^2$ and $-2x^2$, we see that they are opposites, which suggests that one graph would be a reflection of the other across the x-axis.

Unit 7 Lesson 11 Practice Problems

1. Here are four graphs. Match each graph with a quadratic equation that it represents.

Quadratic equation

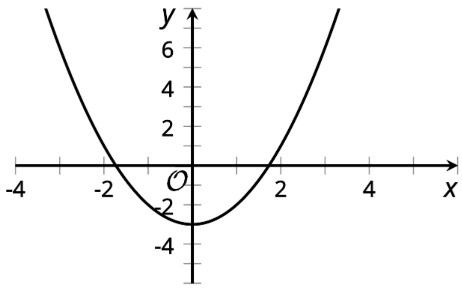
a. $y = x^2$

b. $y = x^2 + 5$

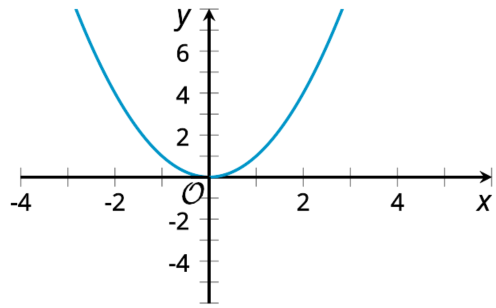
c. $y = x^2 + 7$

d. $y = x^2 - 3$

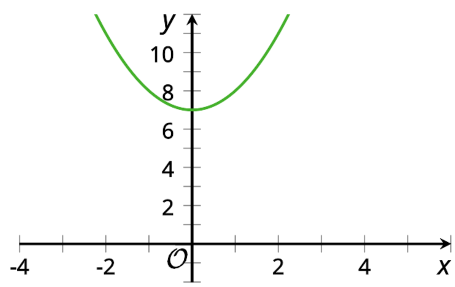
Graph A



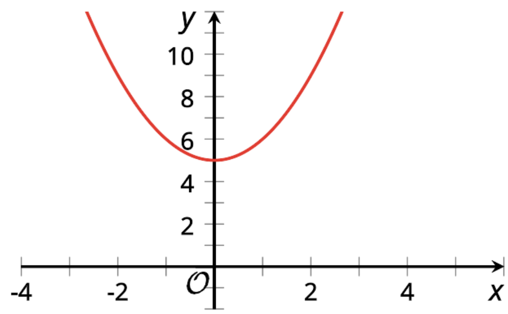
Graph B



Graph C



Graph D



2. The two equations $y = (x + 2)(x + 3)$ and $y = x^2 + 5x + 6$ are equivalent.

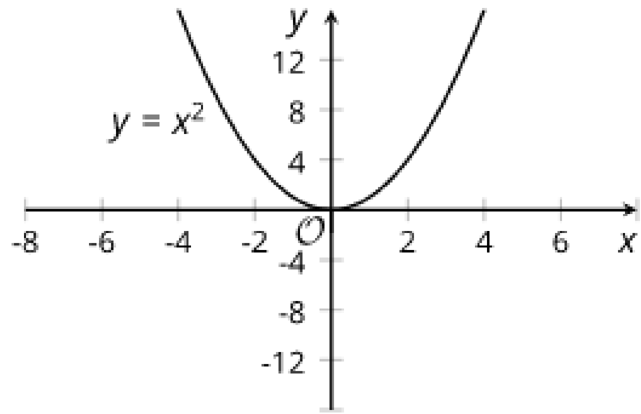
   a. Which equation helps find the x-intercepts most efficiently?

   b. Which equation helps find the y-intercept most efficiently?

3. Here is a graph that represents $y = x^2$.

   On the same coordinate plane, sketch and label the graph that represents each equation:

   a. $y = x^2 - 4$



   b. $y = -x^2 + 5$

4. Select **all** equations whose graphs have a y-intercept with a positive y-coordinate.

   a. $y = x^2 + 3x - 2$

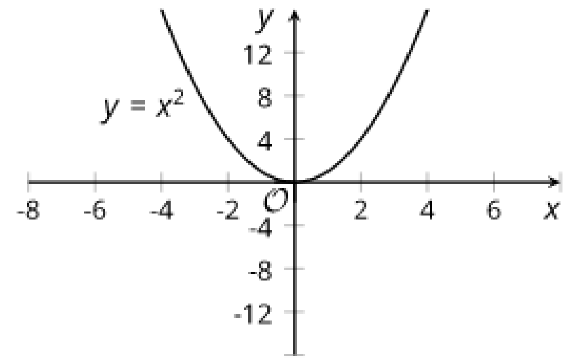
   b. $y = x^2 - 10x$

   c. $y = (x - 1)^2$

   d. $y = 5x^2 - 3x - 5$

   e. $y = (x + 1)(x + 2)$

5. Here is a graph that represents $y = x^2$.



   a. Describe what would happen to the graph if the original equation were modified as follows:

      i. $y = -x^2$

      ii. $y = 3x^2$

      iii. $y = x^2 + 6$

   b. Sketch the graph of the equation $y = -3x^2 + 6$ on the same coordinate plane as $y = x^2$.

6.

a. What are the x-intercepts of the graph that represents $y = (x+1)(x+5)$? Explain how you know.

b. What is the x-coordinate of the vertex of the graph that represents $y = (x+1)(x+5)$? Explain how you know.

c. Find the y-coordinate of the vertex. Show your reasoning.

d. Sketch a graph of $y = (x+1)(x+5)$.

(From Unit 7, Lesson 10)

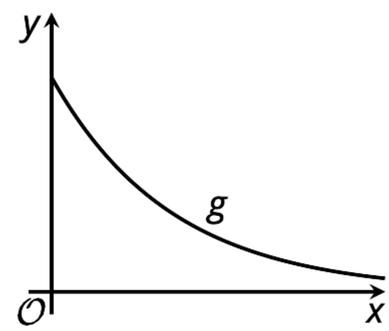
7. Determine the x-intercepts, the vertex, and the y-intercept of the graph of each equation.

Equation	x**-intercepts**	**Vertex**	y**-intercept**
$y = (x-5)(x-3)$			
$y = 2x(8-x)$			

(From Unit 7, Lesson 10)

8. Here is a graph of the function g given by $g(x) = a \cdot b^x$.

   What can you say about the value of b? Explain how you know.



(From Unit 6)

9. Equal amounts of money were invested in stock A and stock B. In the first year, stock A increased in value by 20%, and stock B decreased by 20%. In the second year, stock A decreased in value by 20%, and stock B increased by 20%.

   Was one stock a better investment than the other? Explain your reasoning.

(From Unit 6)

10. The area of a square can be determined by the function $A = s^2$, where A represents the area of the square and s represents the length of one side. Jada is buying square picture frames to hang her high school graduation pictures in her college dorm room.

    a. If the length of one side of the picture frame is s inches, what expression represents the area of the frame?

    b. If Jada decides that she actually needs four picture frames, what expression represents the total area?

    c. If Jada decides to buy one picture frame with a side length 4 times longer than the first, what expression represents its area?

    d. If the length of one side of the original picture frame was 8 inches, what was the area of:

        i. The original picture frame?

        ii. Four of the original picture frames?

        iii. One picture frame with a side length 4 times as long?

(Addressing NC.8.EE.1)

Lesson 12: Graphing the Standard Form (Part Two)

Learning Targets

- I can explain how the values of a and b in $y = ax^2 + bx + c$ affect the graph of the equation.
- I can use patterns to find a formula for the x-coordinate of the vertex of the graph of $y = ax^2 + bx$.

Bridge

In each row, write the equivalent expression. The first row has been done for you as an example.

Factored	Expanded
$-3(5-2y)$	$-15+6y$
$5(a-6)$	
	$6a-2b$
$-4(2w-5z)$	
$-(2x-3y)$	

Warm-up: Equivalent Expressions

1. Complete each row with an equivalent expression in standard form or factored form.
2. What do the quadratic expressions in each column have in common (besides the fact that everything in the left column is in standard form and everything in the right column is in factored form)? Be prepared to share your observations.

Standard form	Factored form
x^2	
	$x(x+9)$
x^2-18x	
	$x(6-x)$
$-x^2+10x$	
	$-x(x+2.75)$

Activity 1: What about the Linear Term?

1. Using graphing technology:

   a. Graph $y = x^2$ and then experiment with adding different linear terms (for example, $x^2 + 4x$, $x^2 + 20x$, $x^2 - 50x$). Record your observations.

   b. Graph $y = -x^2$ and then experiment with adding different linear terms. Record your observations.

2. In the previous question, you should have seen that adding a linear term shifts the vertex both vertically and horizontally from (0,0). Use the structure of each equation to find the coordinates of the vertex without graphing.

Equation	Factored form	x-intercepts	Vertex (x, y)
$y = x^2 + 6x$			
$y = x^2 - 10x$			
$y = -x^2 + 50x$			
$y = -x^2 - 36x$			

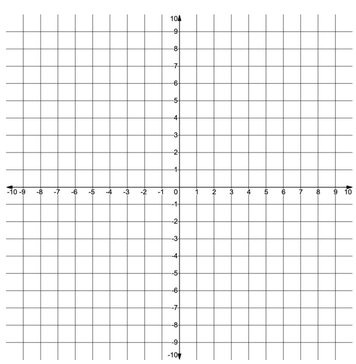
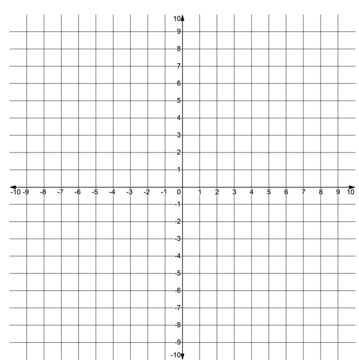
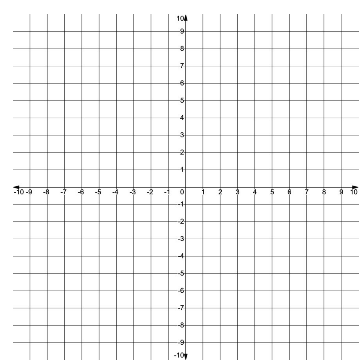
3. Based on what you observed in question 1 and your findings in question 2, make a generalization about how the form $y = ax^2 + bx$ and the x-intercepts and vertex are related.

Activity 2: Graphing $ax^2 + bx$

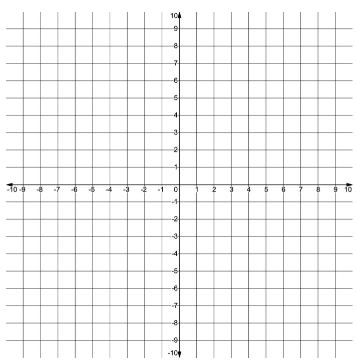
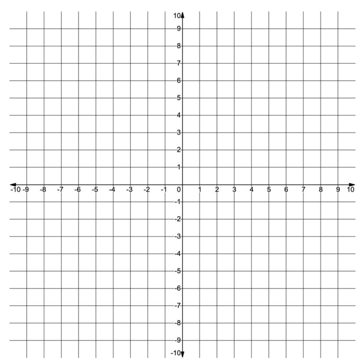
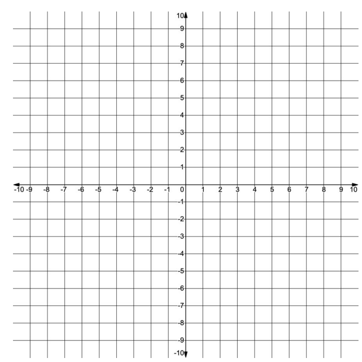
Decide with your partner who will complete the table labeled “Partner A” and who will complete the table labeled “Partner B.”

Complete each column in your table, without technology, based on the information given to you. After sketching each graph, check your graph in Desmos. If correct, place a check in row 6. If incorrect, use row 6 to name how your graph is different from the graph Desmos provides and revise the generalization you made in Activity 1, or to record a question to ask your partner during the discussion that follows.

Partner A

	Column A	**Column B**	**Column C**
Standard form	$y = x^2 + 4x$		
Factored form		$y = -2x(x+6)$	
x-intercepts			$(0,0)$ and $(10,0)$
Vertex (x, y)			$(5, 25)$
Sketch the graph			
Desmos check			

Partner B

	Column A	Column B	Column C
Standard form	$y=-x^2+4x$		
Factored form			$y=-5x(x-2)$
x**-intercepts**		$(0,0)$ and $(12,0)$	
Vertex (x,y)		$(6,-36)$	
Sketch the graph			
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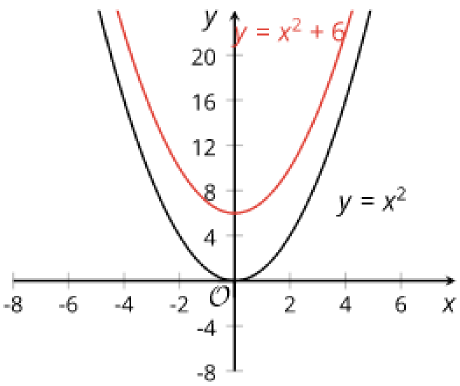
Lesson Debrief

Lesson 12 Summary and Glossary



In an earlier lesson, we saw that a quadratic function written in standard form $ax^2 + bx + c$ can tell us some things about the graph that represents it. The coefficient a can tell us whether the graph of the function opens upward or downward, and also gives us information about whether it is narrow or wide. The constant term c can tell us about its vertical position.

Recall that the graph representing $y = x^2$ is an upward-opening parabola with the vertex at $(0, 0)$. The vertex is also the x-intercept and the y-intercept.

Suppose we add 6 to the squared term: $y = x^2 + 6$. Adding a 6 shifts the graph upwards, so the vertex is at $(0, 6)$. The vertex is the y-intercept and the graph is centered on the y-axis.

What can the linear term bx tell us about the graph representing a quadratic function? When we compare the graphs of $y = x^2$ and $y = x^2 + 6x$, we see that the vertex of the graph is no longer the y-intercept and is not centered on the y-axis. We can calculate the x-coordinate of the vertex exactly by:

- writing $x^2 + 6x$ in factored form: $x(x + 6)$
- finding the x-intercepts: $(-6, 0)$ and $(0, 0)$
- then calculating the number halfway between the x-coordinates: -3.

After we know the x-coordinate of the vertex, we can use the equation, $y = x^2 + 6x$, to find the y-coordinate. $y = (-3)^2 + 6(-3) = -9$. This means the vertex of the graph is $(-3, -9)$.

We can use the same process to find the x-coordinate of the vertex of any parabola with equation $y = ax^2 + bx$:

- factored form: $y = x(ax + b)$
- x-intercepts: $(0, 0)$ and $(\frac{-b}{a}, 0)$ (because if $ax + b = 0$, $x = \frac{-b}{a}$)
- halfway between 0 and $\frac{-b}{a}$ is half of $\frac{-b}{a}$, or $\frac{-b}{2a}$

This means that if we have a quadratic function with equation $y = ax^2 + bx$, we can find the x-coordinate of the vertex of its graph by calculating the value of $\frac{-b}{2a}$. In fact, this method works to find the vertex of any quadratic function with an equation written in standard form. Since the graph of $y = ax^2 + bx + c$ is just an upward or downward shift of the equation $y = ax^2 + bx$, the x-coordinates of the vertex of each graph should be the same.

Unit 7 Lesson 12 Practice Problems

1. Here are four graphs. Match each graph with the quadratic equation that it represents.

Quadratic equation

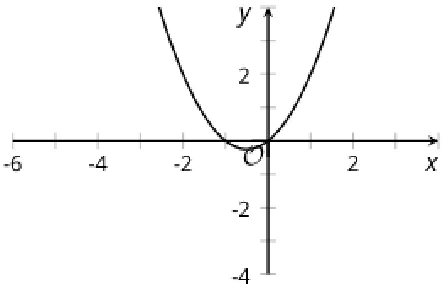
a. $y = x^2 + x$

b. $y = -x^2 + 2$

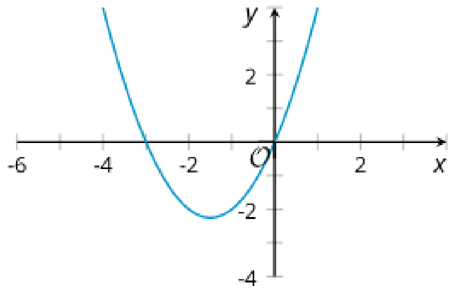
c. $y = x^2 - x$

d. $y = x^2 + 3x$

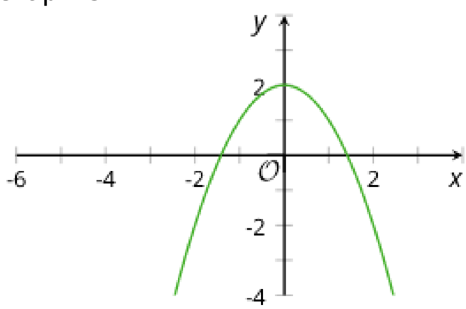
Graph A



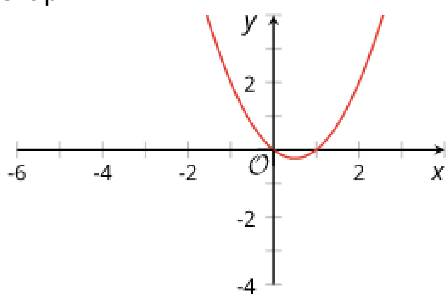
Graph B



Graph C



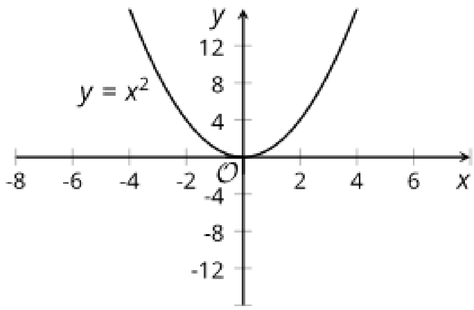
Graph D



2. Complete the table without graphing the equations.

Equation	x-intercepts	Vertex (x, y)
$y = x^2 + 12x$		
$y = x^2 - 3x$		
$y = -x^2 + 16x$		
$y = -x^2 - 24x$		

3. Here is a graph that represents $y = x^2$.



   a. Describe what would happen to the graph if the original equation were changed to $y = x^2 - 6x$. Predict the x- and y-intercepts of the graph and the quadrant where the vertex is located.

   b. Sketch the graph of the equation $y = x^2 - 6x$ on the same coordinate plane as $y = x^2$.

4. Select **all** equations whose graph opens upward.

   a. $y = -x^2 + 9x$

   b. $y = 10x - 5x^2$

   c. $y = (2x - 1)^2$

   d. $y = (1 - x)(2 + x)$

   e. $y = x^2 - 8x - 7$

5. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

   a. $(x + 3)(x + 4)$

   b. $(x + 3)(x + 7)$

   c. $(3x + 4)(x + 3)$

   d. $(x + 7)(3x + 1)$

   1. $x^2 + 10x + 21$

   2. $3x^2 + 13x + 12$

   3. $3x^2 + 22x + 7$

   4. $x^2 + 7x + 12$

(From Unit 7, Lesson 7)

6. A bank loans $4,000 to a customer at a $9\frac{1}{2}\%$ annual interest rate.

   Write an expression to represent how much the customer will owe, in dollars, after 5 years without payment.

(From Unit 6)

7. Determine whether each expression is equivalent to $\frac{1}{x^{-6}}$.

   a. $(x^2)^{-3}$

   b. $\frac{x^9}{x^3}$

   c. $x \cdot x^2 \cdot x^3$

(From Unit 6)

8. Which ordered pair is a solution to this system of equations? $\begin{cases} 7x + 5y = 59 \\ 3x - 9y = 159 \end{cases}$

   a. $(-17, -12)$

   b. $(-17, 12)$

   c. $(17, -12)$

   d. $(17, 12)$

(From Unit 3)

9. The density of an object can be found by dividing its mass by its volume. Write an equation to represent the relationship between the three quantities (density, mass, and volume) in each situation. Let the density, D, be measured in grams/cubic centimeters (or g/cm^3).

   a. The mass is 500 grams, and the volume is 40 cubic centimeters.

   b. The mass is 125 grams, and the volume is v cubic centimeters.

   c. The volume is 1.4 cubic centimeters, and the density is 80 grams per cubic centimeter.

   d. The mass is m grams, and the volume is v cubic centimeters.

(From Unit 2)

10. Complete the equation with numbers that make the expression on the right side of the equal sign equivalent to the expression on the left side.[1]

$$75a + 25b = \_\_\_\_(\_\_\_\_\_\ a + b)$$

(Addressing NC.7.EE.1)

Lesson 13: Graphs That Represent Situations

Learning Targets

- I can explain how a quadratic equation and its graph relate to a situation.

Bridge

From 6 p.m. to midnight, the equation $t = -1.7h + 5.5$ represents the temperature in degrees Celsius, t, after a given number of hours, h, on a cold night in the North Carolina mountains. From 6 a.m. to noon, the equation $C = 1.1h - 2.3$ represents the temperature in degrees Celsius, C, after a given number of hours, h, as the sun rises.

1. Describe what is happening to the temperature based on the slope of each equation.

2. Describe what you know about the temperatures based on the y-intercepts of each equation.

3. Sketch a graph of each equation. How are they the same, and how are they different?

Warm-up: More Pharmaceutical Profiting

The profit of a small pharmaceutical company's insulin, within one week, is modeled by the equation $p(v) = -0.3v^2 + 150v$, where p represents profit in dollars, and v represents the number of vials of insulin sold.

1. Find $p(0)$ and $p(500)$. What do these values mean in terms of the company's profit?

2. How many vials of insulin would the company need to sell to earn the maximum profit? Explain how you know.

Activity 1: A Catapulted Pumpkin

The equation $h = 2 + 23.7t - 4.9t^2$ represents the height of a pumpkin that is catapulted up in the air as a function of time, t, in seconds. The height is measured in meters above ground. The pumpkin is shot up at a vertical velocity of 23.7 meters per second.

1. Without writing anything down, consider these questions:

    a. What do you think the 2 in the equation tells us in this situation? What about the $-4.9t^2$?
    b. If we graph the equation, will the graph open upward or downward? Why?
    c. Where do you think the vertical intercept would be?
    d. What about the horizontal intercepts?
    e. Over what interval do you think the graph will increase? Over what interval do you think the graph will decrease?

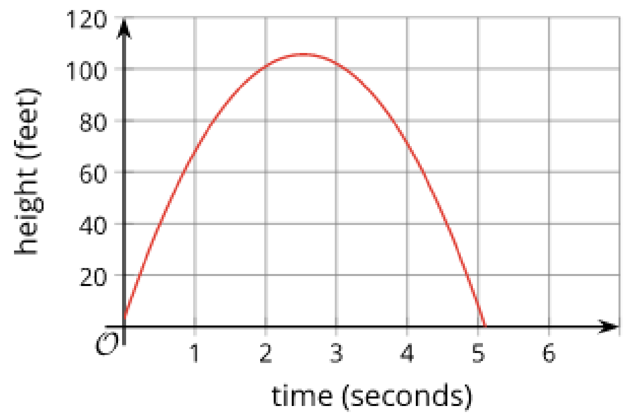
2. Graph the equation using Desmos.

3. Identify the vertical and horizontal intercepts, the vertex of the graph, and where the graph is increasing and decreasing. Explain what each feature of the graph means in this situation.

Are You Ready For More?

What approximate vertical velocity would this pumpkin need for it stay in the air for about 10 seconds? (Assume that it is still shot from 2 meters in the air and that the effect of gravity pulling it down is the same.)

Activity 2: Flight of Two Baseballs



Here is a graph that represents the height of a baseball, h, in feet as a function of time, t, in seconds after it was hit by Player A.

The function g defined by $g(t) = (-16t - 1)(t - 4)$ represents the height in feet of a baseball t seconds after it was hit by Player B. Without graphing function g, answer the following questions and explain or show how you know.

1. Which player's baseball stayed in flight longer?

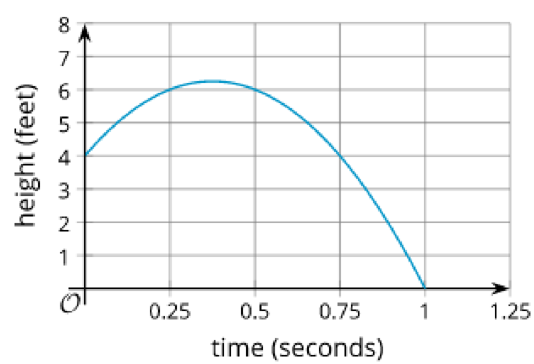
2. Which player's baseball reached a greater maximum height?

3. How can you find the height at which each baseball was hit?

Lesson Debrief

Lesson 13 Summary and Glossary

Let's say a tennis ball is hit straight up in the air, and its height in feet above the ground is modeled by the equation $f(t) = 4 + 12t - 16t^2$. Here is a graph that represents the function, from the time the tennis ball was hit until the time it reached the ground.



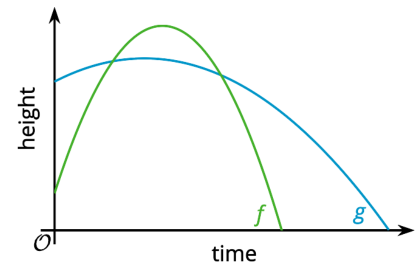
In the graph, we can see some information we already know, and some new information:

- The 4 in the equation means the graph of the function intersects the vertical axis at 4. It shows that the tennis ball was 4 feet off the ground at $t = 0$, when it was hit.
- The horizontal intercept is $(1, 0)$. It tells us that the tennis ball hits the ground 1 second after it was hit.
- The vertex of the graph is at approximately $(0.4, 6.3)$. This means that about 0.4 second after the ball was hit, it reached the maximum height of about 6.3 feet.

The equation can be written in factored form as $f(t) = (-16t - 4)(t - 1)$. From this form, we can see that the zeros of the function are $t = 1$ and $t = -\frac{1}{4}$. The negative zero, $-\frac{1}{4}$, is not meaningful in this situation, because the time before the ball was hit is irrelevant.

Unit 7 Lesson 13 Practice Problems

1. Here are graphs of functions f and g.



   Each represents the height of an object being launched into the air as a function of time.

   a. Which object was launched from a higher point?

   b. Which object reached a higher point?

   c. Which object was launched with the greater upward velocity?

   d. Which object landed last?

2. (*Technology required.*) The function h given by $h(t) = (1 - t)(8 + 16t)$ models the height of a ball in feet, t seconds after it was thrown.

   a. Find the zeros of the function. Show or explain your reasoning.

   b. What do the zeros tell us in this situation? Are both zeros meaningful?

   c. From what height is the ball thrown? Explain your reasoning.

   d. About when does the ball reach its highest point, and about how high does the ball go? Show or explain your reasoning.

3. The height in feet of a thrown football is modeled by the equation $f(t) = 6 + 30t - 16t^2$, where time, t, is measured in seconds.

    a. What does the constant 6 mean in this situation?

    b. What does the $30t$ mean in this situation?

    c. How do you think the squared term $-16t^2$ affects the value of the function f? What does this term reveal about the situation?

4. The height in feet of an arrow is modeled by the equation $h(t) = (1 + 2t)(18 - 8t)$, where t is time in seconds after the arrow is shot.

    a. When does the arrow hit the ground? Explain or show your reasoning.

    b. From what height is the arrow shot? Explain or show your reasoning.

5. The height in feet of a soccer ball is modeled by the equation $g(t) = 2 + 50t - 16t^2$, where t is time measured in seconds after it was kicked.

   a. How far above the ground was the ball when kicked?

   b. What was the initial upward velocity of the ball?

   c. Why is the coefficient of the squared term negative?

6. Two objects are launched into the air. In both functions, t is time in seconds after launch.

   - The height, in feet, of object A is given by the equation $f(t) = 4 + 32t - 16t^2$
   - The height, in feet, of the object B is given by the equation $g(t) = 2.5 + 40t - 16t^2$.

   a. Which object was launched from a greater height? Explain how you know.

   b. Which object was launched with a greater upward velocity? Explain how you know.

7. (*Technology required.*) Consider the following questions:

   a. Predict the x- and y-intercepts of the graph of the quadratic function defined by the expression $(x+6)(x-6)$. Explain how you made your predictions.

   b. Check your predictions by graphing $y=(x+6)(x-6)$.

(From Unit 7, Lesson 9)

8. (*Technology required.*) The functions f and g are given by $f(x)=13x+6$ and $g(x)=0.1\cdot(1.4)^x$.

   a. Which function eventually grows faster, f or g? Explain how you know.

   b. Use graphing technology to decide when the graphs of f and g meet.

(From Unit 6)

9. (*Technology required.*) A student needs to get a loan of $12,000 for the first year of college. Bank A has an annual interest rate of 5.75%, bank B has an annual interest rate of 7.81%, and bank C has an annual rate of 4.45%.

    a. If we graph the amount owed for each loan as a function of years without payment, predict what the three graphs would look like. Describe or sketch your prediction.

    b. Use graphing technology to plot the graph of each loan balance.

    c. Based on your graph, how much would the student owe for each loan when they graduate from college in four years?

    d. Based on your graph, if no payments are made, how much would the student owe for each loan after 10 years?

(From Unit 6)

10. According to the University of Michigan Department of Medicine, the average weight of a baby in its first 4 days of life can be modeled by the equation $w = -2d + 120$, where w represents the weight in ounces and d represents the number of days after the baby is born. Then, the equation $w = 0.7d + 112$ models an average baby's weight for the next 30 days of its life, where w represents the weight in ounces and d represents the number of days after day 4 of the baby's life.

    a. Describe what is happening to the weight based on the slope of each equation.

    b. Describe what you know about the weights based on the y-intercepts of each equation.

    c. Describe the graphs of each equation in words. How are they the same, and how are they different?

(Addressing NC.8.F.4)

Lessons 14 & 15: Checkpoint

Learning Targets

- I can add and subtract linear and quadratic expressions.
- I can continue to grow as a mathematician and challenge myself.
- I can share what I know mathematically.

Station B: Adding and Subtracting Quadratics

Follow your teacher's directions to access this station's Desmos activity. Use the available space below to show your work.

Station D: Solving Number Riddles and Puzzles

1. List all the pairs of integers whose product is 12.

    a. Circle any pairs with a sum of 7.

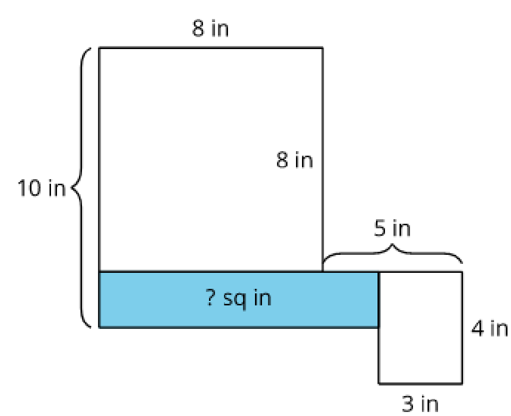
    b. Draw a box around any pairs with a sum of 13.

2. Here is a riddle: “I have 2 dogs. The product of their ages is 12, and the sum of their ages is 8. How old are my dogs?"

3. Here is a harder riddle: “I have 3 daughters. The product of their ages is 24. The sum of their ages is the lowest number it could possibly be. How old are they?”

4. A mathematician threw a party. She told her guests, “I have a riddle for you. I have three daughters. The product of their ages is 72. The sum of their ages is the same as my house number. How old are my daughters?” The guests went outside to look at the house number. They thought for a few minutes, and then said, “This riddle can’t be solved!” The mathematician said, “Oh yes, I forgot to tell you the last clue. My youngest daughter prefers strawberry ice cream.” With this last clue, the guests could solve the riddle. How old are the mathematician’s daughters?

5. Can you find the missing area in the figure below? Share your reasoning.



Station E: Fair Living Wage[1]

You will be given a set of Fair Wage cooperative cards highlighting multiple types of families in Charlotte. You will see their hourly wage information and more. Your goal is to use mathematics to decide whether or not you think six families in Charlotte are paid fair wages.

As a team, you will:

- Draw graphs showing the relationships between the number of hours work and the total wages for multiple families
- Use a different colored pencil or marker for each family
- Identify the dependent and independent variables
- Analyze the wage and rent data

Use this data in the table below on rental prices in Charlotte metro to complete your task:[2]

Studio	1BDR	2BDR	3BDR	4BDR
$653	$745	$864	$1,173	$1,469

1. Each member of the group should select a Fair Wage cooperative card.
2. Draw a graph and write an equation for each family's earnings over time using a colored pencil or marker based on the color named on the card.

3. Determine the amount each family will need to work in a month to afford monthly rent for an apartment based on their family size (assume there are 4 weeks in a month).

[1] Adapted from Frances Harper
[2] U.S. Department of Housing and Urban Development Office of Policy Development and Research. *Hud User*. https://www.huduser.gov

After all members have graphed and solved an equation, answer the following questions:

4. Which family needs to work the **fewest** hours per month to pay rent? How do you know?

5. Which family needs to work the **most** hours per month to pay rent? How do you know?

6. Were there any lines that intersect? If so, what does that intersection mean in context of this problem?

7. Financial advisors recommend that you only use 30% of your monthly income to pay for rent. (The rest of your income goes to taxes, clothing, food, transportation, savings, and other expenses.) Does the number of hours needed to pay rent match 30% of the earnings for each family? If not, what kind of housing can the family afford with only 30% of their income?

8. According to the National Low Income Housing Coalition,[3] in 2021, a family in NC needs to make $18.46 per hour to afford a moderate two-bedroom home. How well does this match what you found? Explain your reasoning.

[3] https://reports.nlihc.org/oor

Station F: Micro-Modeling

1. An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Is more than one option available? Explain your reasoning.[4]

[4] Adapted from Achievethecore.org

2. Mai has a new rabbit that she named Wascal. She wants to build Wascal a pen so that the rabbit has space to move around safely. Mai has purchased a 72-foot roll of fencing to build a rectangular pen.[5]

   a. If Mai uses the whole roll of fencing, what are some of the possible dimensions of the pen?

   b. If Mai wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

   c. Write and display a model for the area of the rectangular pen in terms of the length of one side.

   d. What kind of function is this? How do you know?

   e. Bonus: If the rabbit pen does not need to be a rectangle, is there a way to get more area with 72 feet of fencing around the perimeter?

[5] Adapted from Open Up Resources. Access the full curriculum, supporting tools, and educator communities at openupresources.org.

Station G: Reflection

Describe your year in mathematics so far. You can do this through any form of writing or drawing. Feel free to include your experiences with peers, math material, teachers, etc.

Lesson 16: Finding Unknown Inputs

Learning Targets

- I can explain the meaning of a solution to an equation in terms of a situation.
- I can write a quadratic equation that represents a situation involving geometric measures.

Bridge

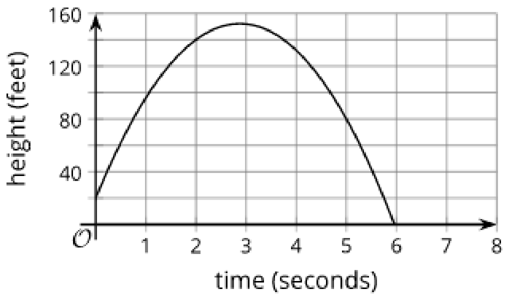
The picture frame shown, made of tree bark, can display an 8" by 10" photo. The width of the bark is 1.5”. How many linear inches of tree bark are needed to create this frame?

Warm-up: What Goes Up Must Come Down

A mechanical device is used to launch a potato vertically into the air. The potato is launched from a platform 20 feet above the ground, with an initial vertical velocity of 92 feet per second.

The function $h(t) = -16t^2 + 92t + 20$ models the height of the potato over the ground, in feet, t seconds after launch.

Here is the graph representing the function.



For each question, be prepared to explain your reasoning.

1. What is the height of the potato 1 second after launch?

2. 8 seconds after launch, will the potato still be in the air?

3. Will the potato reach 120 feet? If so, when will it happen?

4. When will the potato hit the ground?

Activity 1: A Trip to the Frame Shop[1]

Your teacher will give you a picture that is 7 inches by 4 inches, a piece of framing material measuring 4 inches by 2.5 inches, and a pair of scissors.

Cut the framing material to create a rectangular frame for the picture. The frame should have the same thickness all the way around and have no overlaps. All of the framing material should be used (with no leftover pieces). Framing material is very expensive!

You get three copies of the framing material in case you make mistakes and need to recut.

Are You Ready For More?

Han says, “The perimeter of the picture is 22 inches. If I cut the framing material into 9 pieces, each one being 2.5 inches by $\frac{4}{9}$ inch, I’ll have more than enough material to surround the picture because those pieces would mean 22.5 inches for the frame.”

Do you agree with Han? Explain your reasoning.

[1] This activity was inspired by the post “When I Got Them to Beg” by Fawn Nguyen on http://fawnnguyen.com/got-beg/ and used with permission.

Activity 2: Representing the Framing Problem

Here is a diagram that shows the picture with a frame that is the same thickness all the way around. The picture is 7 inches by 4 inches. The frame is created from 10 square inches of framing material (in the form of a rectangle measuring 4 inches by 2.5 inches).

1. Write an equation to represent the relationship between the outer measurements of the picture and of the frame, and the area of the framed picture. Be prepared to explain what each part of your equation represents.

2. What would a solution to this equation mean in this situation?

Lesson Debrief

Lesson 16 Summary and Glossary

The height of a softball, in feet, t seconds after someone throws it straight up, can be defined by $f(t) = -16t^2 + 32t + 5$. The input of function f is time, and the output is height.

We can find the output of this function at any given input. For instance:

- At the beginning of the softball's journey, when $t = 0$, its height is given by $f(0)$.
- Two seconds later, when $t = 2$, its height is given by $f(2)$.

The values of $f(0)$ and $f(2)$ can be found using a graph or by evaluating the expression $-16t^2 + 32t + 5$ at those values of t.

What if we know the output of the function and want to find the inputs? For example:

- When does the softball hit the ground?

  Answering this question means finding the values of t that make $f(t) = 0$, or solving $-16t^2 + 32t + 5 = 0$.

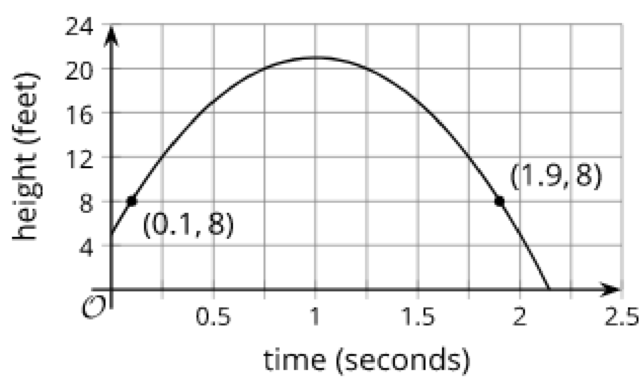
- How long will it take the ball to reach 8 feet?

  This means finding one or more values of t that make $f(t) = 8$, or solving the equation $-16t^2 + 32t + 5 = 8$.

The equations $-16t^2 + 32t + 5 = 0$ and $-16t^2 + 32t + 5 = 8$ are quadratic equations. One way to solve these equations is by graphing $y = f(t)$.

- To answer the first question, we can look for the horizontal intercepts of the graph, where the vertical coordinate is 0.
- To answer the second question, we can look for the horizontal coordinates that correspond to a vertical coordinate of 8.

We can see that there are two solutions to the equation $-16t^2 + 32t + 5 = 8$.



The softball has a height of 8 feet twice, when going up and when coming down, and these occur when t is about 0.1 or 1.9.

Often, when we are modeling a situation mathematically, an approximate solution is good enough. Sometimes, however, we would like to know exact solutions, and it may not be possible to find them using a graph.

In this unit, we will learn more about quadratic equations and how to solve them exactly using algebraic techniques.

Unit 7 Lesson 16 Practice Problems

1. Jada throws a paper airplane from her treehouse. The height of the plane is a function of time and can be modeled by the equation $h(t) = 25 + 2.5t - \frac{1}{2}t^2$. Height is measured in feet and time is measured in seconds.

    a. Evaluate $h(0)$ and explain what this value means in this situation.

    b. What would a solution to $h(t) = 0$ mean in this situation?

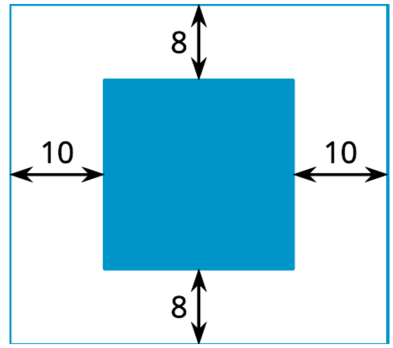
    c. What does the equation $h(9) = 7$ mean?

    d. What does the model say about the airplane 2.5 seconds after Jada throws it if each of these statements is true?

    $h(2) = 28$ $\quad$ $h(2.5) = 28.125$ $\quad$ $h(3) = 28$

2. A garden designer designed a square decorative pool. The pool is surrounded by a walkway.

   On two opposite sides of the pool, the walkway is 8 feet. On the other two opposite sides, the walkway is 10 feet. Here is a diagram of the design.

   The final design for the pool and walkway covers a total area of 1,440 square feet.

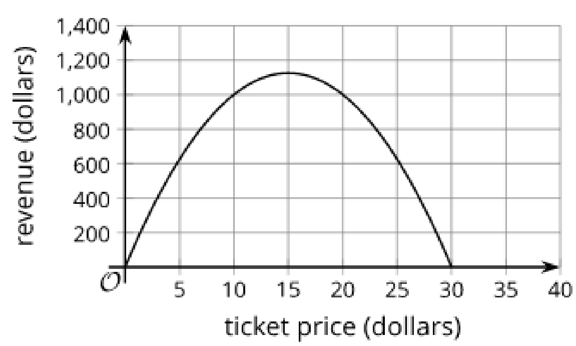
   a. The side length of the square pool is x. Write an expression that represents:

      - the total length of the rectangle (including the pool and walkway)
      - the total width of the rectangle (including the pool and walkway)
      - the total area of the pool and walkway

   b. Write an equation of the form: $\textit{your expression} = 1,440$. What does a solution to the equation mean in this situation?

3. The revenue from a youth league baseball game depends on the price of per ticket, x.

   Here is a graph that represents the revenue function, R.

   Select **all** the true statements.

   a. $R(5)$ is a little more than 600.

   b. $R(600)$ is a little less than 5.

   c. The maximum possible ticket price is $15.

   d. The maximum possible revenue is about $1,125.

   e. If tickets cost $10, the predicted revenue is $1,000.

   f. If tickets cost $20, the predicted revenue is $1,000.

4. A square picture has a frame that is 3 inches thick all the way around. The total side length of the picture and frame is x inches.

   Which expression represents the area of the square picture, without the frame? If you get stuck, try sketching a diagram.

   a. $(2x+3)(2x+3)$

   b. $(x+6)(x+6)$

   c. $(2x-3)(2x-3)$

   d. $(x-6)(x-6)$

5. Add or subtract:

   a. $(7x^2-3x+2.4)+(-4x^2-5x-4)$

   b. $(8x^2+7)-(3x^2-5.5x+11.5)$

   c. $(4x^2-2x)+(9x^2+5)-(8.2x-4.5)$

(From Unit 7, Lessons 14 and 15)

6. Two rocks are launched straight up in the air. The height of rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is measured in seconds, and height is measured in feet.

   a. Which rock is launched from a higher point?

   b. Which rock is launched with greater velocity?

(From Unit 7, Lesson 5)

7. A bacteria population is 10,000 when it is first measured and then doubles each day.

   a. Use this information to complete the table.

   b. Which is the first day, after the population was originally measured, that the bacteria population is more than 1,000,000?

d, time (days)	p, population (thousands)
0	
1	
2	
5	
10	
d	

   c. Write an equation relating p, the bacteria population, to d, the number of days since it was first measured.

(From Unit 6)

8. An American traveler who is heading to Europe is exchanging some U.S. dollars for European euros. At the time of his travel, 1 dollar can be exchanged for 0.91 euros.

   a. Find the amount of money in euros that the American traveler would get if he exchanged 100 dollars.

   b. What if he exchanged 500 dollars?

   c. Write an equation that gives the amount of money in euros, e, as a function of the dollar amount being exchanged, d.

   d. Upon returning to America, the traveler has 42 euros to exchange back into U.S. dollars. How many dollars would he get if the exchange rate is still the same?

   e. Write an equation that gives the amount of money in dollars, d, as a function of the euro amount being exchanged, e.

(From Unit 5)

9. Suppose m and c each represent the position number of a letter in the alphabet, but m represents the letters in the original message and c represents the letters in a secret code. The equation $c = m + 2$ is used to encode a message.

    a. Write an equation that can be used to decode the secret code into the original message.

    b. What does this code say: "OCVJ KU HWP!"?

(From Unit 5)

10. Give a value for r, the correlation coefficient, that would indicate that a line of best fit has a positive slope and models the data well.

(From Unit 4)

Lesson 17: When and Why Do We Write Quadratic Equations?

Learning Targets

- I can use a graph to find the solutions to a quadratic equation but also know its limitations.
- I can recognize the factored form of a quadratic expression and know when it can be useful for solving problems.
- I can write quadratic equations and reason about their solutions in terms of their situation.

Bridge

Use the function $f(x) = -5x - 10$ to answer the questions.

1. What is the y-intercept of the graph of $y = f(x)$?
2. What is the x-intercept of the graph of $y = f(x)$?
3. Solve the equation $0 = -5x - 10$. Which intercept does this reveal? Explain.
4. Find $f(0)$. What intercept does this reveal? Explain.

Warm-up: How Many Tickets?

The expression $12t + 2.50$ represents the cost to purchase tickets for a play, where t is the number of tickets. Be prepared to explain your response to each question.

1. A family paid \$62.50 for tickets. How many tickets were bought?

2. A teacher paid \$278.50 for tickets for her students. How many tickets were bought?

Activity 1: The Flying Potato Again

Earlier in this unit, you saw an equation that defines the height of a potato as a function of time after it was launched from a mechanical device. Here is a different function modeling the height of a potato, in feet, t seconds after being fired from a different device:

$$f(t) = -16t^2 + 80t + 64$$

1. What equation would we solve to find the time at which the potato hits the ground?

2. Use any method to find a solution to this equation.

Activity 2: Revenue from Ticket Sales

The expressions $p(200 - 5p)$ and $-5p^2 + 200p$ define the same function. The function models the revenue a school would earn from selling raffle tickets at p dollars each.

1. At what price or prices would the school collect \$0 revenue from raffle sales? Explain or show your reasoning.

2. The school staff noticed that there are two ticket prices that would both result in a revenue of \$500. How would you find out what those two prices are?

Are You Ready For More?

Can you find the following prices without graphing?

1. If the school charges $10, it will collect $1,500 in revenue. Find another price that would generate $1,500 in revenue.

2. If the school charges $28, it will collect $1,680 in revenue. Find another price that would generate $1,680 in revenue.

3. Find the price that would produce the maximum possible revenue. Explain your reasoning.

Lesson Debrief

Lesson 17 Summary and Glossary

The height of a potato that is launched from a mechanical device can be modeled by a function g. Here are two expressions that are equivalent and both define function g.

$$-16x^2+80x+96$$

$$-16(x-6)(x+1)$$

Notice that one expression is in standard form and the other is in factored form.

Suppose we wish to know, without graphing the function, the time when the potato will hit the ground. We know that the value of the function at that time is 0, so we can write:

$$-16x^2+80x+96=0$$

$$-16(x-6)(x+1)=0$$

Let's try solving $-16x^2+80x+96=0$, using some familiar moves. For example:

- Subtract 96 from each side: $-16x^2+80x=-96$
- Apply the distributive property to rewrite the expression on the left: $-16(x^2-5x)=-96$
- Divide both sides by -16: $x^2-5x=6$
- Apply the distributive property to rewrite the expression on the left: $x(x-5)=6$

These steps don't seem to get us any closer to a solution. We need some new moves!

What if we use the other equation? Can we find the solutions to $-16(x-6)(x+1)=0$?

Earlier, we learned that the *zeros* of a quadratic function can be identified when the expression defining the function is in factored form. The solutions to $-16(x-6)(x+1)=0$ are the zeros of function g, so this form may be more helpful! We can reason that:

- If x is 6, then the value of $x-6$ is 0, so the entire expression has a value of 0.
- If x is -1, then the value of $x+1$ is 0, so the entire expression also has a value of 0.

This tells us that 6 and -1 are solutions to the equation, and that the potato hits the ground after 6 seconds. (A negative value of time is not meaningful, so we can disregard the -1.)

Both equations we see here are **quadratic equations**. In general, a quadratic equation is an equation that can be expressed as $ax^2+bx+c=0$.

In upcoming lessons, we will learn how to rewrite quadratic equations into forms that make the solutions easy to see.

Quadratic equation: An equation that is equivalent to one of the form ax^2+bx+c, where a, b, and c are constants and $a \neq 0$.

Unit 7 Lesson 17 Practice Problems

1. Select **all** values of x that are solutions to the equation $(x-5)(7x-21)=0$.

    a. -7

    b. -5

    c. -3

    d. 0

    e. 3

    f. 5

    g. 7

2. The expressions $30x^2-105x-60$ and $(5x-20)(6x+3)$ define the same function, f.

    a. Which expression makes it easier to find $f(0)$? Explain your reasoning.

    b. Find $f(0)$.

    c. Which expression makes it easier to find the values of x that make the equation $f(x)=0$ true? Explain or show your reasoning.

    d. Find the values of x that make $f(x)=0$.

3. A band is traveling to a new city to perform a concert. The revenue from their ticket sales is a function of the ticket price, x, and can be modeled with $(x-6)(250-5x)$.

   What are the ticket prices at which the band would make no money at all?

First number	Second number	Product
1	14	14
3	12	36
5	10	50
7	8	56

4. Here are a few pairs of positive numbers whose sums are 15. The pair of numbers that has a sum of 15 and will produce the largest possible product is **not** shown.

   Find this pair of numbers.

First number	Second number	Product
1	14	14
3	12	36
5	10	50
7	8	56

5. Two students built a small rocket from a kit and attached an altimeter (a device for recording altitude or height) to the rocket. They use the table to record the height of the rocket over time since it is launched, based on the data from the altimeter.

Time (seconds)	0	1	3	4	7	8
Height (meters)	0	110.25	236.25	252	110.25	0

Function h gives the height in meters as a function of time in seconds, t.

a. What is the value of $h(3)$?

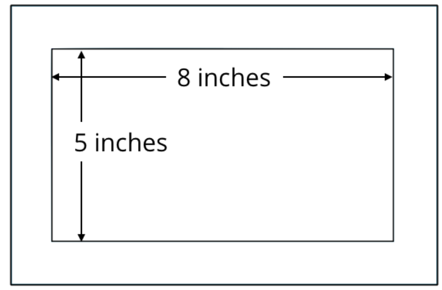
b. What value of t gives $h(t) = 252$?

c. Explain why $h(0) = h(8)$.

d. Based on the data, which equation about the function could be true: $h(2) = 189$ or $h(189) = 2$? Explain your reasoning.

(From Unit 7, Lesson 16)

6. The screen of a tablet has dimensions 8 inches by 5 inches. The border around the screen has thickness x.



   a. Write an expression for the total area of the tablet, including the frame.

   b. Write an equation for which your expression is equal to 50.3125. Explain what a solution to this equation means in this situation.

   c. Try to find the solution to the equation. If you get stuck, try guessing and checking. It may help to think about tablets that you have seen.

(From Unit 7, Lesson 16)

7. (*Technology required.*) Two objects are launched upward. Each function gives the distance from the ground in meters as a function of time, t, in seconds.

   Object A: $f(t) = 25 + 20t - 5t^2$

   Object B: $g(t) = 30 + 10t - 5t^2$

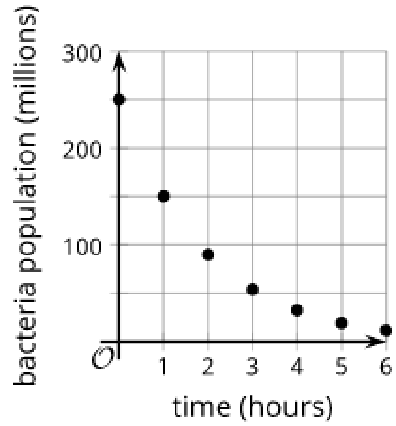
   Use graphing technology to graph each function.

   a. Which object reaches the ground first? Explain how you know.

   b. What is the maximum height of each object?

(From Unit 7, Lesson 5)

8. The graph shows a bacteria population decreasing exponentially over time.

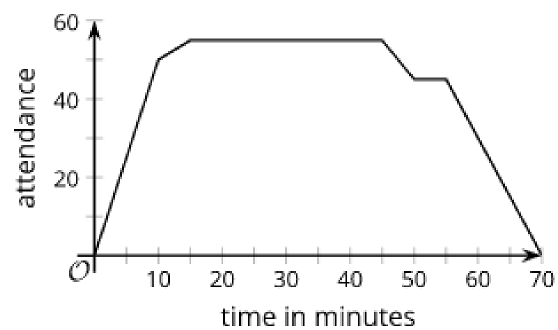


The equation $p = 100,000,000 \cdot \left(\frac{2}{3}\right)^h$ gives the size of a second population of bacteria, where h is the number of hours since it was measured at 100 million.

Which bacterial population decays more quickly? Explain how you know.

(From Unit 6)

9. The graph shows the attendance at a sports game as a function of time in minutes.



   a. Describe how attendance changed over time.

   b. Describe the domain.

   c. Describe the range.

(From Unit 5)

10. A set of kitchen containers can be stacked to save space. The height of the stack is given by the expression $1.5c + 7.6$, where c is the number of containers.

    a. Find the height of a stack made of eight containers.

    b. A tower made of all the containers is 40.6 cm tall. How many containers are in the set?

    c. Noah looks at the equation and says, “7.6 must be the height of a single container.” Do you agree with Noah? Explain your reasoning.

(From Unit 2)

11. Use the function $f(x) = -2x + 5$ to answer the questions.

    a. What is the y-intercept of the graph of $y = f(x)$?

    b. What is the x-intercept of the graph of $y = f(x)$?

    c. Solve the equation $0 = -2x + 5$. Which intercept does this reveal? Explain.

    d. Find $f(0)$. What intercept does this reveal? Explain.

(Building On NC.8.F.4)

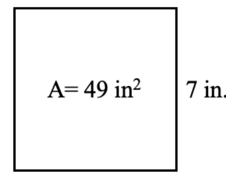
Lesson 18: Solving Quadratic Equations by Reasoning

Learning Targets

- I can find solutions to quadratic equations by reasoning about the values that make the equation true.
- I know that quadratic equations may have two solutions.

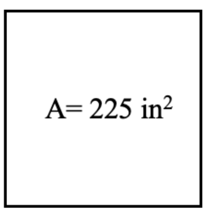
Bridge

Square A has an area of 49 square inches, and each side has a length of 7 inches, because the square root of 49, written $\sqrt{49} = 7$, and $7^2 = 49$.

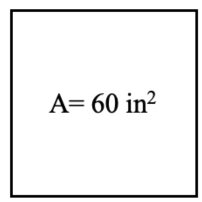


Use square root notation to write the length of one side of each square below:

1.



2.



Warm-up: How Many Solutions?

How many solutions does each equation have? What are the solution(s)? Be prepared to explain how you know.

1. $x^2 = 9$
2. $x^2 = 0$
3. $x^2 - 1 = 3$
4. $2x^2 = 50$
5. $(x+1)(x+1) = 0$
6. $x(x-6) = 0$
7. $(x-1)(x-1) = 4$

Activity 1: Finding Pairs of Solutions

Each of these equations has two solutions. What are they? Explain or show your reasoning.

1. $n^2 + 4 = 404$
2. $432 = 3n^2$
3. $144 = (n+1)^2$
4. $(n-5)^2 - 30 = 70$

Are You Ready For More?

1. How many solutions does the equation $(x-3)(x+1)(x+5)=0$ have? What are the solutions?

2. How many solutions does the equation $(x-2)(x-7)(x-2)=0$ have? What are the solutions?

3. Write a new equation that has 10 solutions.

Activity 2: Employing Square Roots

Follow your teacher's directions to access the Desmos activity. Use the available space below to show your work.

Lesson Debrief

Lesson 18 Summary and Glossary

Some quadratic equations can be solved by performing the same operations to each side of the equal sign and reasoning about values of the variable would make the equation true.

Suppose we wanted to solve $7x^2 = 112$. We can proceed like this:

- Divide each side by 7: $x^2 = 16$
- What number can be squared to get 16? $?^2 = 16$
- Using the fact that $\sqrt{16} = 4$, we know that there are two numbers that when squared result in 16, 4 and -4: $4^2 = 16$ and $(-4)^2 = 16$
- Therefore, $x = 4$ and $x = -4$

This means that both $x = 4$ and $x = -4$ make the equation true and are solutions to the equation.

Suppose we wanted to solve $3(x+1)^2 - 75 = 0$. We can proceed like this:

- Add 75 to each side: $3(x+1)^2 = 75$
- Divide each side by 3: $(x+1)^2 = 25$
- What number can be squared to get 25? $?^2 = 25$
- Using the fact that $\sqrt{25} = 5$, we know that there are two numbers that when squared result in 25, 5 and -5: $5^2 = 25$ and $(-5)^2 = 25$
- If $x + 1 = 5$, then $x = 4$.
- If $x + 1 = -5$, then $x = -6$.

This means that both $x = 4$ and $x = -6$ make the equation true and are solutions to the equation.

Unit 7 Lesson 18 Practice Problems

1. Consider the equation $x^2 = 9$.
   a. Show that 3 and -3 are each a solution to the equation.

   b. Show that 9 and $\sqrt{3}$ are each **not** a solution to the equation.

2. Solve $(x-1)^2 = 16$. Explain or show your reasoning.

3. Here is one way to solve the equation $\frac{5}{9}y^2 = 5$. Explain what is done in each step.

$$\begin{array}{ll} \frac{5}{9}y^2 = 5 & \textit{Original equation} \\ 5y^2 = 45 & \textit{Step 1} \\ y^2 = 9 & \textit{Step 2} \\ y = 3 \text{ or } y = -3 & \textit{Step 3} \end{array}$$

4. Diego and Jada are working together to solve the quadratic equation $(x-2)^2 = 100$.

   Diego solves the equation by dividing each side of the equation by 2 and then adding 2 to each side. He writes:

   $$(x-2) = 50$$
   $$x = 52$$

   Jada asks Diego why he divides each side by 2 and he says, “I want to find a number that equals 100 when multiplied by itself. That number is half of 100.”

   a. What mistake is Diego making?

   b. If you were Jada, what could you say to Diego to help him realize his mistake?

5. As part of a publicity stunt (an event designed to draw attention), a TV host drops a watermelon from the top of a tall building. The height of the watermelon t seconds after it is dropped is given by the function $h(t) = 850 - 16t^2$, where h is in feet.

   a. Find $h(4)$. Explain what this value means in this situation.

   b. Find $h(0)$. What does this value tell us about the situation?

   c. Is the watermelon still in the air 8 seconds after it is dropped? Explain how you know.

(From Unit 7, Lesson 16)

6. Add or subtract:

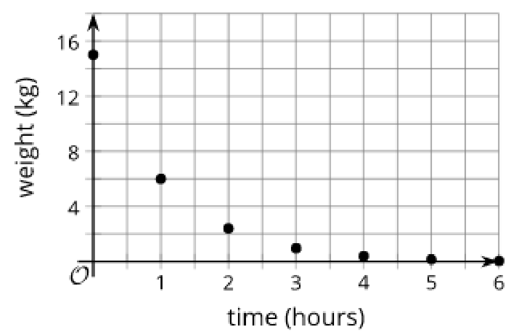
   a. $(-16t^2 - 32t + 14) + (t^2 + 14t - 32)$

   b. $(-5x - 6) - (-0.2x^2 - 4x + 11)$

   c. $(25x^2 - 16) + (16x^2 - 9)$

(From Unit 7, Lessons 14 and 15)

7. The graph shows the weight of snow as it melts. The weight decreases exponentially.



   a. By what factor does the weight of the snow decrease each hour? Explain how you know.

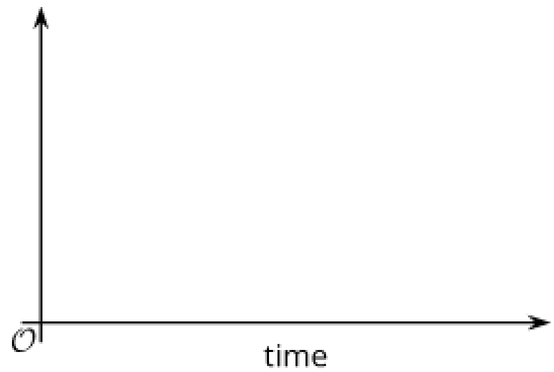
   b. Does the graph predict that the weight of the snow will reach 0? Explain your reasoning.

   c. Will the weight of the actual snow, represented by the graph, reach 0? Explain how you know.

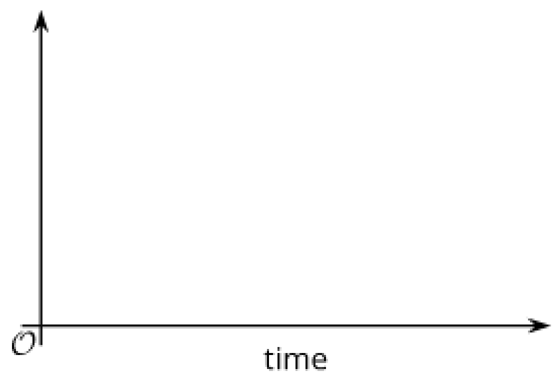
(From Unit 6)

8. Sketch a graph to represent each quantity described as a function of time. Be sure to label the vertical axis.

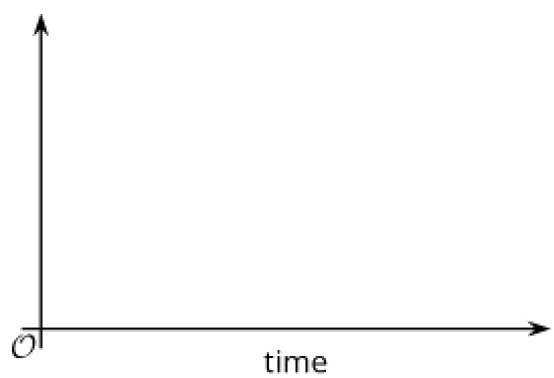
   a. Swing: The height of your feet above the ground while swinging on a swing on a swing at a playground



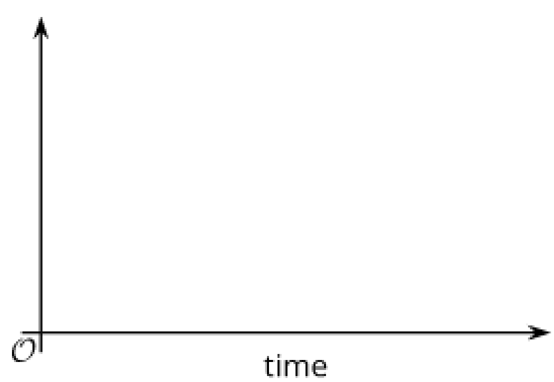
   b. Slide: The height of your shoes above ground as you walk to a slide, go up a ladder, and then go down a slide



   c. Merry-go-round: Your distance from the center of a merry-go round as you ride the merry-go-round



   d. Merry-go-round, again: Your distance from your friend, who is standing next to the merry-go-round as you go around



(From Unit 5)

9. A zoo offers unlimited drink refills to visitors who purchase its souvenir cup. The cup and the first fill cost \$10, and refills after that are \$2 each. The expression $10+2r$ represents the total cost of the cup and r refills.

   a. A family visited the zoo several times over a summer. That summer, they paid \$30 for one cup and multiple refills. How many refills did they buy?

   b. A visitor has \$18 to spend on drinks at the zoo today and buys a souvenir cup. How many refills can they afford during the visit?

   c. Another visitor spent \$10 on this deal. Did they buy any refills? Explain how you know.

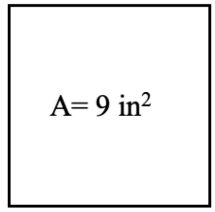
(From Unit 2)

10. Solve the equation: $\frac{2}{3}(12x-30)=5x+2$

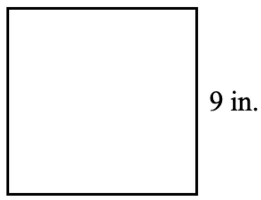
(From Unit 2)

11.

a. What is the length of one side of the square?



b. What is the area of the square?



c. Explain how square roots and powers of 2 helped you figure out the side length and area.

(Addressing NC.8.EE.2)

Lesson 19: Solving Quadratic Equations with the Zero Product Property

Learning Targets

- I can find solutions to quadratic equations when one side is a product of factors and the other side is zero.
- I can explain the meaning of the “zero product property.”

Warm-up: Solve These Equations

What values of the variables make each equation true?

1. $6+2a=0$
2. $7b=0$
3. $7(c-5)=0$
4. $g \cdot h=0$

Activity 1: Take the Zero Product Property Out for a Spin

For each equation, find its solution or solutions. Be prepared to explain your reasoning.

1. $x-3=0$	2. $x+11=0$
3. $2x+11=0$	4. $x(2x+11)=0$
5. $(x-3)(x+11)=0$	6. $(x-3)(2x+11)=0$

Are You Ready For More?

1. Use factors of 48 to find as many whole-number solutions as you can to the equation $(x-3)(x+5)=48$.

2. Once you think you have all the solutions, explain why these must be the only solutions.

Activity 2: Revisiting a Projectile

We have seen quadratic functions modeling the height of a projectile as a function of time.

Here are two ways to define the same function that approximates the height of a projectile in meters, t seconds after launch:

$h(t)=-5t^2+27t+18$ $\qquad$ $h(t)=(-5t-3)(t-6)$

1. Which way of defining the function allows us to use the zero product property to find out when the height of the object is 0 meters?

2. Without graphing, determine at what time the height of the object is 0 meters. Show your reasoning.

Lesson Debrief

Lesson 19 Summary and Glossary

The **zero product property** says that if the product of two numbers is 0, then one of the numbers must be 0. In other words, if $a \cdot b = 0$, then either $a = 0$ or $b = 0$. This property is handy when an equation we want to solve states that the product of two factors is 0.

Suppose we want to solve $m(m+9) = 0$. This equation says that the product of m and $(m+9)$ is 0. For this to be true, either $m = 0$ or $m + 9 = 0$, so both 0 and -9 are solutions.

Here is another equation: $(u - 2.345)(14u + 2) = 0$. The equation says the product of $(u - 2.345)$ and $(14u + 2)$ is 0, so we can use the zero product property to help us find the values of u. For the equation to be true, one of the factors must be 0.

- For $u - 2.345 = 0$ to be true, u would have to be 2.345.
- For $14u + 2 = 0$ or $14u = -2$ to be true, u would have to be $-\frac{2}{14}$ or $-\frac{1}{7}$.

The solutions are 2.345 and $-\frac{1}{7}$.

In general, when a quadratic expression in factored form is on one side of an equation and 0 is on the other side, we can use the zero product property to find its solutions.

Zero product property: If the product of two numbers is 0, then one of the numbers must be 0.

Unit 7 Lesson 19 Practice Problems

1. If the equation $(x+10)x=0$ is true, which statement is also true according to the zero product property?

    a. only $x=0$

    b. either $x=0$ or $x+10=0$

    c. either $x^2=0$ or $10x=0$

    d. only $x+10=0$

2. What are the solutions to the equation $(10-x)(3x-9)=0$?

    a. -10 and 3

    b. -10 and 9

    c. 10 and 3

    d. 10 and 9

3. Solve each equation.

    a. $(x-6)(x+5)=0$

    b. $(x-3)(\frac{2}{3}x-6)=0$

    c. $(-3x-15)(x+7)=0$

4. Consider the expressions $(x-4)(3x-6)$ and $3x^2-18x+24$.

   Show that the two expressions define the same function.

5. Kiran saw that if the equation $(x+2)(x-4)=0$ is true, then, by the zero product property, either $x+2$ is 0 or $x-4$ is 0. He then reasoned that, if $(x+2)(x-4)=72$ is true, then either $x+2$ is equal to 72 or $x-4$ is equal to 72.

   Explain why Kiran's conclusion is incorrect.

6. Andre wants to solve the equation $5x^2-4x-18=20$. He uses a graphing calculator to graph $y=5x^2-4x-18$ and $y=20$ and finds that the graphs cross at the points $(-2.39, 20)$ and $(3.19, 20)$.

   a. Substitute each x-value Andre found into the expression $5x^2-4x-18$. Then evaluate the expression.

   b. Why did neither solution make $5x^2-4x-18$ equal exactly 20?

(From Unit 7, Lesson 17)

7. Select **all** the solutions to the equation $7x^2 = 343$.

    a. 49

    b. $-\sqrt{7}$

    c. 7

    d. -7

    e. $\sqrt{49}$

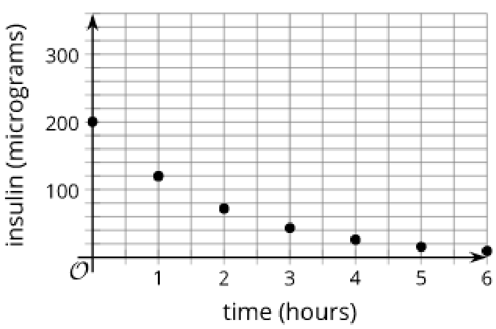
    f. $\sqrt{-49}$

    g. $-\sqrt{49}$

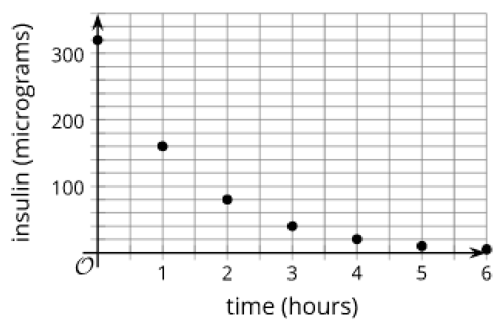
(From Unit 7, Lesson 18)

8. Here are two graphs that correspond to two patients, A and B. Each graph shows the amount of insulin, in micrograms (mcg) in a patient's body h hours after receiving an injection. The amount of insulin in each patient decreases exponentially.

Patient A



Patient B



Select **all** statements that are true about the insulin level of the two patients.

a. After the injection, the patients have the same amount of insulin in their bodies.

b. An equation for the micrograms of insulin, a, in Patient A's body h hours after the injection is $a = 200 \cdot (\frac{3}{5})^h$.

c. The insulin in Patient A is decaying at a faster rate than in Patient B.

d. After 3 hours, Patient A has more insulin in their body than Patient B.

e. At some time between 2 and 3 hours, the patients have the same insulin level.

(From Unit 6)

9. Scientists are trying to invent a new kind of milk container that will help the environment by decaying faster in landfills. One sample material decays at a rate represented by the function $w(m) = 2.3 \cdot 0.87^m$, where $w(m)$ represents the remaining weight of the milk container in ounces and m represents the number of months since the carton was manufactured.

    a. What percentage of the container decays each month?

    b. What was the initial weight of the milk container?

(From Unit 5)

10.

    a. Write the equation of a line parallel to the line $x = 6$ through point $(-2, -3)$.

    b. Write the equation of a line perpendicular to the line $x = 6$ through point $(-2, -3)$.

(From Unit 3)

Lesson 20: How Many Solutions?

Learning Targets

- I know that quadratic equations can have no solutions and can explain why there are none.
- I can describe the relationship between the solutions to quadratic equations and the graph of the related function.
- I can explain why dividing by a variable to solve a quadratic equation is not a good strategy.

Bridge

Figure out what value of x makes each equation true, and explain how you figured out each value.

1. $3x = 27$
2. $x^3 = 27$
3. $3x = 24$
4. $x^3 = 24$
5. Why do you need different operations to determine each value?

Warm-up: Four Equations

Decide whether each statement is true or false. Explain your reasoning.

Statement	True or False?	Explanation
1. 3 is the only solution to $x^2 - 9 = 0$		
2. A solution to $x^2 + 25 = 0$ is -5		
3. $x(x-7) = 0$ has two solutions		
4. 5 and -7 are the solutions to $(x-5)(x+7) = 12$		

Activity 1: Solving by Graphing

Han is solving three equations by graphing.

$(x-5)(x-3)=0$ $\quad$ $(x-5)(x-3)=-1$ $\quad$ $(x-5)(x-3)=-4$

1. To solve the first equation, $(x-5)(x-3)=0$, he graphed $y=(x-5)(x-3)$ and then looked for the x-intercepts of the graph.
   a. Explain why the x-intercepts can be used to solve $(x-5)(x-3)=0$.

   b. What are the solutions?

2. To solve the second equation, Han rewrote it as $(x-5)(x-3)+1=0$. He then graphed $y=(x-5)(x-3)+1$.

   Use graphing technology to graph $y=(x-5)(x-3)+1$. Then, use the graph to solve the equation. Be prepared to explain how you use the graph for solving.

3. Solve the third equation using Han's strategy.

4. Think about the strategy you used and the solutions you found.

   a. Why might it be helpful to rearrange each equation to equal 0 on one side and then graph the expression on the non-zero side?

   b. How many solutions does each of the three equations have?

Are You Ready For More?

The equations $(x-3)(x-5)=-1$, $(x-3)(x-5)=0$, and $(x-3)(x-5)=3$ all have whole-number solutions.

1. Use graphing technology to graph each of the following pairs of equations on the same coordinate plane. Analyze the graphs and explain how each pair helps to solve the related equation.

   a. $y=(x-3)(x-5)$ and $y=-1$

   b. $y=(x-3)(x-5)$ and $y=0$

   c. $y=(x-3)(x-5)$ and $y=3$

2. Use the graphs to help you find a few other equations of the form $(x-3)(x-5)=z$ that have whole-number solutions.

3. Find a pattern in the values of z that give whole-number solutions.

4. Without solving, determine if $(x-5)(x-3)=120$ and $(x-5)(x-3)=399$ have whole-number solutions. Explain your reasoning.

Activity 2: Finding All the Solutions

Solve each equation. Be prepared to explain or show your reasoning.

1. $x^2 = 121$
2. $x^2 - 31 = 5$
3. $(x - 4)(x - 4) = 0$
4. $(x + 3)(x - 1) = 5$
5. $(x + 1)^2 = -4$
6. $(x - 4)(x - 1) = 990$

Activity 3: Analyzing Errors in Equation Solving

1. Consider $(x-5)(x+1)=7$. Priya reasons that if this is true, then either $x-5=7$ or $x+1=7$. So, the solutions to the original equation are 12 and 6.

   Do you agree? If not, where was the mistake in Priya's reasoning?

2. Consider $x^2-10x=0$. Diego says to solve we can just divide each side by x to get $x-10=0$, so the solution is 10. Mai says, "I wrote the expression on the left in factored form, which gives $x(x-10)=0$, and ended up with two solutions: 0 and 10."

   Do you agree with either strategy? Explain your reasoning.

Lesson Debrief

Lesson 20 Summary and Glossary

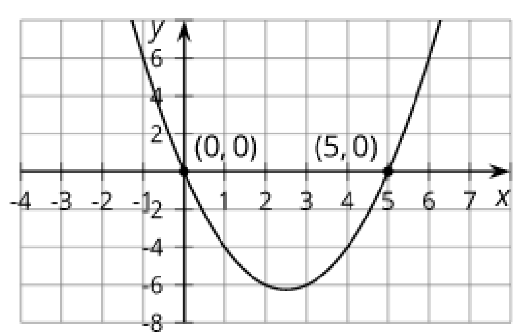
Quadratic equations can have two solutions, one solution, or no real solutions.

We can find out how many real solutions a quadratic equation has and what the solutions are by rearranging the equation into the form of $expression = 0$, graphing the function that the expression defines, and determining its zeros. Here are some examples.

- $x^2 = 5x$

  Let's first subtract $5x$ from each side and rewrite the equation as $x^2 - 5x = 0$. We can think of solving this equation as finding the zeros of a function defined by $x^2 - 5x$.

  If the output of this function is y, we can graph $y = x^2 - 5x$ and identify where the graph intersects the x-axis, where the y-coordinate is 0.

  From the graph, we can see that the x-intercepts are $(0, 0)$ and $(5, 0)$, so $x^2 - 5x$ equals 0 when x is 0 and when x is 5.

  The graph readily shows that there are two solutions to the equation.

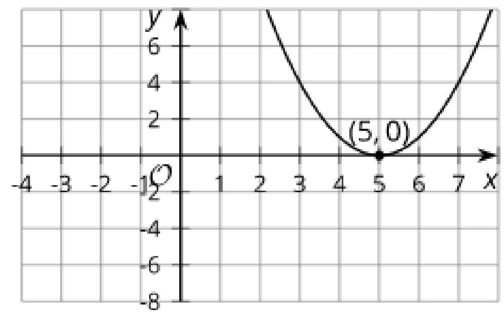
Note that the equation $x^2 = 5x$ can be solved without graphing, but we need to be careful *not* to divide both sides by x. Doing so will give us $x = 5$ but will show no trace of the other solution, $x = 0$!

Even though dividing both sides by the same value is usually acceptable for solving equations, we avoid dividing by the same variable because it may eliminate a solution.

- $(x-6)(x-4) = -1$

  Let's rewrite the equation as $(x-6)(x-4)+1=0$ and consider it to represent a function defined by $(x-6)(x-4)+1$ and whose output, y, is 0.

  Let's graph $y=(x-6)(x-4)+1$ and identify the x-intercepts.

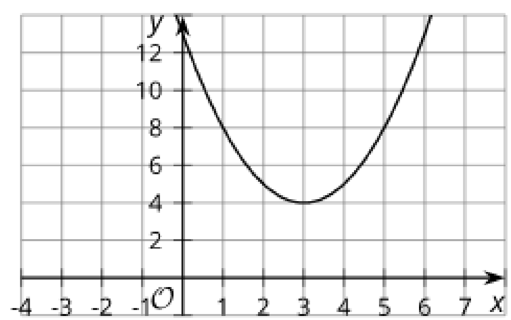
  The graph shows one x-intercept at $(5, 0)$. This tells us that the function defined by $(x-6)(x-4)+1$ has only one zero.

  It also means that the equation $(x-6)(x-4)+1=0$ is true only when $x=5$. The value 5 is the only solution to the equation.

- $(x-3)(x-3) = -4$

  Rearranging the equation gives $(x-3)(x-3)+4=0$.

  Let's graph $y=(x-3)(x-3)+4$ and find the x-intercepts.

  The graph does not intersect the x-axis, so there are no x-intercepts.

  This means there are no real number x-values that can make the expression $(x-3)(x-3)+4$ equal 0, so the function defined by $y=(x-3)(x-3)+4$ has no zeros.

  The equation $(x-3)(x-3)=-4$ has no real number solutions.

  We can see that this is the case even without graphing. $(x-3)(x-3)=-4$ is equivalent to $(x-3)^2=-4$. Because no real number can be squared to get a negative value, the equation has no real number solutions.

Earlier you learned that graphing is not always reliable for showing precise solutions. This is still true here. The x-intercepts of a graph are not always whole-number values. While they can give us an idea of how many solutions there are and what the values may be (at least approximately), for exact solutions we still need to rely on algebraic ways of solving.

Unit 7 Lesson 20 Practice Problems

1. Rewrite each equation so that the expression on one side could be graphed and the x-intercepts of the graph would show the solutions to the equation.

   a. $3x^2 = 81$

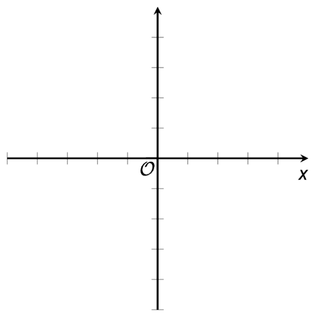
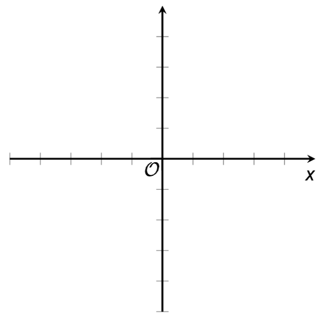
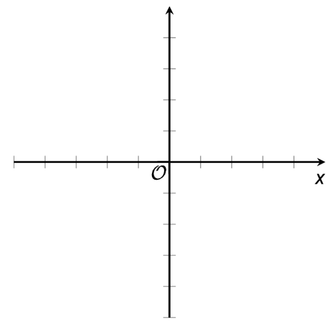
   b. $(x-1)(x+1) - 9 = 5x$

   c. $x^2 - 9x + 10 = 32$

   d. $6x(x-8) = 29$

2.

a. Here are equations that define quadratic functions f, g, and h. Sketch a graph, by hand or using technology, that represents each equation. Indicate the vertex, intercepts, and general shape and direction of the graph.

$f(x) = x^2 + 4$	$g(x) = x(x + 3)$	$h(x) = (x - 1)^2$
		

b. Determine how many real number solutions each $f(x) = 0$, $g(x) = 0$, and $h(x) = 0$ has. Explain how you know.

3. Mai is solving the equation $(x - 5)^2 = 0$. She writes that the solutions are $x = 5$ and $x = -5$ Han looks at her work and disagrees. He says that only $x = 5$ is a solution. Who do you agree with? Explain your reasoning.

4. Decide if each equation has 0, 1, or 2 solutions and explain how you know.

   a. $x^2 - 144 = 0$

   b. $x^2 + 144 = 0$

   c. $x(x - 5) = 0$

   d. $(x - 8)^2 = 0$

   e. $(x + 3)(x + 7) = 0$

5. If the equation $(x - 4)(x + 6) = 0$ is true, which is also true according to the zero product property?

   a. only $x - 4 = 0$

   b. only $x + 6 = 0$

   c. $x - 4 = 0$ or $x + 6 = 0$

   d. $x = -4$ or $x = 6$

(From Unit 7, Lesson 19)

6.

a. Solve the equation $25 = 4z^2$.

b. Show that your solution or solutions are correct.

(From Unit 7, Lesson 18)

7. To solve the quadratic equation $3(x-4)^2 = 27$, Andre and Clare wrote the following:

Andre

$3(x-4)^2 = 27$
$(x-4)^2 = 9$
$x^2 - 4^2 = 9$
$x^2 - 16 = 9$
$x^2 = 25$
$x = 5$ or $x = -5$

Clare

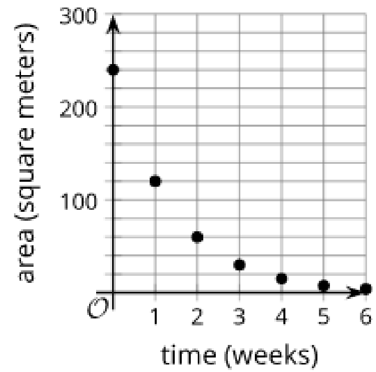
$3(x-4)^2 = 27$
$(x-4)^2 = 9$
$x - 4 = 3$
$x = 7$

a. Identify the mistake(s) each student made.

b. Solve the equation and show your reasoning.

(From Unit 7, Lesson 18)

8. The graph shows the number of square meters, A, covered by algae in a lake w weeks after it was first measured.



In a second lake, the number of square meters, B, covered by algae is defined by the equation $B = 975 \cdot (\frac{2}{5})^w$, where w is the number of weeks since it was first measured.

For which algae population is the area decreasing more rapidly? Explain how you know.

(From Unit 6)

9. At the end of each year, 10% interest is charged on a \$500 loan. The interest applies to any unpaid balance on the loan, including previous interest.

Select **all** the expressions that represent the loan balance after two years if no payments are made.

a. $500 + 2 \cdot (0.1) \cdot 500$

b. $500 \cdot (1.1) \cdot (1.1)$

c. $500 + (0.1) + (0.1)$

d. $500 \cdot (1.1)^2$

e. $(500 + 50) \cdot (1.1)$

(From Unit 6)

10. For the following pairs of equations, determine whether x is greater than, less than, or equal to y.

Fill in the blank with >, <, or =.

a. $9x = 27$; $y^3 = 27$ x ____ y

b. $3x = 50$; $y^3 = 50$ x ____ y

c. $5x = 100$; $y^5 = 100$ x____ y

(Addressing NC.8.EE.2)

Lesson 21: Rewriting Quadratic Expressions in Factored Form (Part One)

Learning Targets

- I can explain how the numbers in a quadratic expression in factored form relate to the numbers in an equivalent expression in standard form.
- When given quadratic expressions in factored form, I can rewrite them in standard form.
- When given quadratic expressions in the form of $x^2 + bx + c$, I can rewrite them in factored form.

Bridge

Example: Find two numbers that multiply to equal 21 and add to equal 10.

Solution: 7 and 3, because $7 \cdot 3 = 21$ and $7 + 3 = 10$

Now, try some as a game with your classmates and see who can figure out the most without a calculator! For each of the following questions, figure out two numbers that:

a. Multiply to equal 20 and add to equal 12

b. Multiply to equal -6 and add to equal 1

c. Multiply to equal 15 and add to equal -8

d. Multiply to equal 36 and add to equal 15

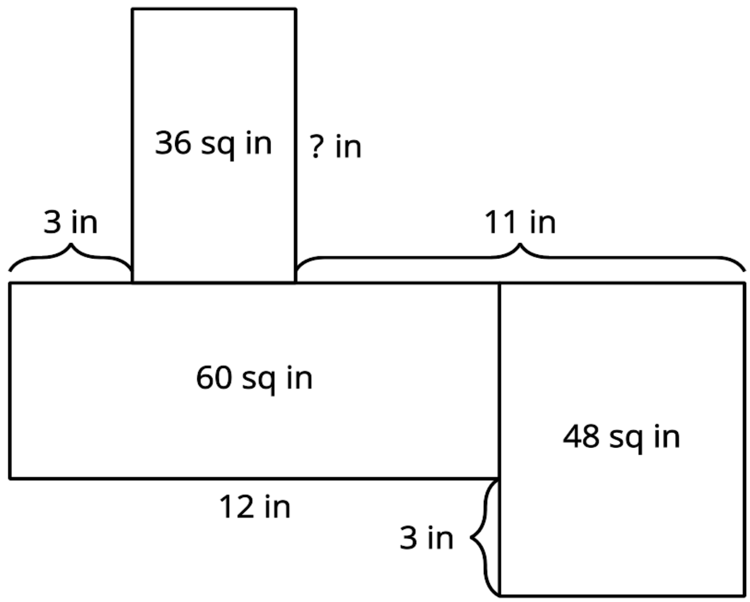
e. Multiply to equal 12 and add to equal 13

f. Multiply to equal -24 and add to equal 5

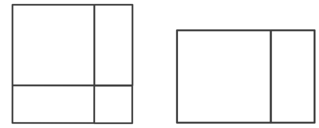
g. Multiply to equal 2 and add to equal -3

Warm-up: Puzzles of Rectangles

Here is a puzzle that involves side lengths and areas of rectangles. Can you find the missing length? Be prepared to explain your reasoning.



Activity 1: Using Diagrams to Understand Equivalent Expressions



1. Use a diagram to show that each pair of expressions is equivalent. Use a diagram similar to the ones shown here to show your models.

   a. $x(x+3)$ and x^2+3x

   b. $x(x+-6)$ and x^2-6x

   c. $(x+2)(x+4)$ and x^2+6x+8

   d. $(x+4)(x+10)$ and $x^2+14x+40$

   e. $(x+-5)(x+-1)$ and x^2-6x+5

   f. $(x-1)(x-7)$ and x^2-8x+7

2. Observe the pairs of expressions from above that involve the product of two sums or two differences. How is each expression in factored form related to the equivalent expression in standard form?

Activity 2: Let's Rewrite Some Expressions!

Each row in the table contains a pair of equivalent expressions.

Complete the table with the missing expressions. If you get stuck, consider drawing a diagram.

Factored form	Standard form
$x(x+7)$	
	x^2+9x
	x^2-8x
$(x+6)(x+2)$	
	$x^2+13x+12$
$(x-6)(x-2)$	
	$x^2-7x+12$
	x^2+6x+9
	$x^2+10x+9$
	$x^2-10x+9$
	x^2-6x+9
	$x^2+(m+n)x+mn$

Lesson Debrief

Lesson 21 Summary and Glossary

Previously, you learned how to expand a quadratic expression in factored form and write it in standard form by applying the distributive property. For example, to expand $(x+4)(x+5)$, we apply the distributive property to multiply x by $(x+5)$ and 4 by $(x+5)$. Then, we apply the property again to multiply x by x and x by 5, and multiply 4 by x and 4 by 5.

To keep track of all the products, we could make a diagram like this:

	x	4
x		
5		

Next, we could write the products of each pair inside the spaces:

	x	4
x	x^2	$4x$
5	$5x$	$4 \cdot 5$

The diagram helps us see that $(x+4)(x+5)$ is equivalent to $x^2+5x+4x+4 \cdot 5$, or in standard form, $x^2+9x+20$.

- The linear term, $9x$, has a coefficient of 9, which is the sum of 5 and 4.
- The constant term, 20, is the product of 5 and 4.

We can use these observations to reason in the other direction: to start with an expression in standard form and write it in factored form. For example, suppose we wish to write $x^2-11x+24$ in factored form.

Let's start by creating a diagram and writing in the terms x^2 and 24. We need to think of two numbers that multiply to make 24 and add up to -11.

	x	
x	x^2	
		24

After some thinking, we see that -8 and -3 meet these conditions. The product of -8 and -3 is 24. The sum of -8 and -3 is -11.

	x	-8
x	x^2	$-8x$
-3	$-3x$	24

So, $x^2-11x+24$ written in factored form is $(x-8)(x-3)$.

Unit 7 Lesson 21 Practice Problems

1. Find two numbers that satisfy the requirements. If you get stuck, try listing all the factors of the first number.

   a. Find two numbers that multiply to 17 and add to 18.

   b. Find two numbers that multiply to 20 and add to 9.

   c. Find two numbers that multiply to 11 and add to -12.

   d. Find two numbers that multiply to 36 and add to -20.

2. Select **all** expressions that are equivalent to $x - 5$.

   a. $x + (-5)$

   b. $x - (-5)$

   c. $-5 + x$

   d. $-5 - x$

   e. $5 - x$

   f. $-5 - (-x)$

   g. $5 + x$

3. Use the diagram to show that:

   a. $(x+4)(x+2)$ is equivalent to x^2+6x+8.

	x	2
x		
4		

   b. $(x-10)(x-3)$ is equivalent to $x^2-13x+30$.

	x	-10
x		
-3		

4. Here are pairs of equivalent expressions—one in standard form and the other in factored form. Find the missing numbers.

   a. $x^2+\boxed{}\,x+\boxed{}$ and $(x-9)(x-3)$

   b. $x^2+12x+32$ and $(x+4)(x+\boxed{})$

   c. $x^2-12x+35$ and $(x-4)(x+\boxed{})$

   d. $x^2-9x+20$ and $(x-4)(x+\boxed{})$

5. (*Technology required.*) When solving the equation $(2-x)(x+1)=11$, Priya graphs $y=(2-x)(x+1)-11$ and then looks to find where the graph crosses the x-axis.

   Tyler looks at Priya's work and says that graphing is unnecessary, and Priya can set up the equations $2-x=11$ and $x+1=11$, so the solutions are $x=-9$ or $x=10$.

   a. Do you agree with Tyler? If so, explain why. If not, where is the mistake in their reasoning?

   b. How many solutions does the equation have? Find out by graphing Priya's equation.

   (From Unit 7, Lesson 20)

6. Find all the values for the variable that make each equation true.

   a. $b(b-4.5)=0$

   b. $(7x+14)(7x+14)=0$

   c. $(2x+4)(x-4)=0$

   d. $(-2+u)(3-u)=0$

   (From Unit 7, Lesson 19)

7. From 2005 to 2015, a population of p lions is modeled by the equation $p = 1,500 \cdot (0.98)^t$, where t is the number of years since 2005.

    a. About how many lions were there in 2005?

    b. Describe what is happening to the population of lions over this decade.

    c. About how many lions are there in 2015? Show your reasoning.

(From Unit 6)

8. Lin charges $5.50 per hour to babysit. The amount of money earned, in dollars, is a function of the number of hours that she babysits.

    Which of the following inputs is impossible for this function?

    a. -1

    b. 2

    c. 5

    d. 8

(From Unit 5)

9. Diego's goal is to walk more than 70,000 steps this week. The mean number of steps that Diego walked during the first 4 days of this week is 8,019.

   a. Write an inequality that expresses the mean number of steps that Diego needs to walk during the last 3 days of this week to walk more than 70,000 steps. Remember to define any variables that you use.

   b. If the mean number of steps Diego walks during the last 3 days of the week is 12,642, will Diego reach his goal of walking more that 70,000 steps this week?

(From Unit 5)

10. A median of a triangle is a segment drawn from one vertex to the midpoint of the opposite side. For Triangle ABC, with $A(6, 10)$, $B(-3, 7)$, and $C(5, 3)$, how long is the median drawn from A to the midpoint of BC?

(From Unit 3)

Lesson 22: Rewriting Quadratic Expressions in Factored Form (Part Two)

Learning Targets

- When given a quadratic expression given in standard form with a negative constant term, I can write an equivalent expression in factored form.
- I can explain how the numbers and signs in a quadratic expression in factored form relate to the numbers and signs in an equivalent expression in standard form.

Warm-up: Sums and Products

1. The product of the integers 2 and -6 is -12. List all the other pairs of integers whose product is -12.

2. Of the pairs of factors you found, list all pairs that have a positive sum. Explain why they all have a positive sum.

3. Of the pairs of factors you found, list all pairs that have a negative sum. Explain why they all have a negative sum.

Activity 1: Negative Constant Terms

1. These expressions are like the ones we have seen before. Each row has a pair of equivalent expressions.

   Complete the table. If you get stuck, consider drawing a diagram.

Factored form	Standard form
$(x+5)(x+6)$	
	$x^2+13x+30$
$(x-3)(x-6)$	
	$x^2-11x+18$

2. Looking at the completed table for question 1 and the table below in question 3, what do you notice and wonder?

3. These expressions are in some ways unlike the ones we have seen before. Each row has a pair of equivalent expressions.

   Complete the table. If you get stuck, consider drawing a diagram.

Factored form	Standard form
$(x+12)(x-3)$	
	$x^2-9x-36$
	$x^2-35x-36$
	$x^2+35x-36$

4. Name some ways that the expressions in the second table are different from those in the first table (aside from the fact that the expressions use different numbers).

Activity 2: Factors of 100 and -100

1. Consider the expression $x^2 + bx + 100$.

   Complete the first table with all pairs of factors of 100 that would give positive values of b and the second table with factors that would give negative values of b.

   For each pair, state the b value they produce. (Use as many rows as needed.)

Positive value of b

Factor 1	Factor 2	b (positive)

Negative value of b

Factor 1	Factor 2	b (negative)

2. Consider the expression $x^2 + bx - 100$.

Complete the first table with all pairs of factors of -100 that would result in positive values of b, the second table with factors that would result in negative values of b, and the third table with factors that would result in a zero value of b.

For each pair of factors, state the b value they produce. (Use as many rows as there are pairs of factors. You may not need all the rows.)

Positive value of b

Factor 1	Factor 2	b (positive)

Negative value of b

Factor 1	Factor 2	b (negative)

Zero value of b

Factor 1	Factor 2	b (zero)

3. Write each expression in factored form:

a. $x^2 - 25x + 100$

b. $x^2 + 15x - 100$

c. $x^2 - 15x - 100$

d. $x^2 + 99x - 100$

Are You Ready For More?

How many different integers b can you find so that the expression $x^2 + 10x + b$ can be written in factored form?

Lesson Debrief

Lesson 22 Summary and Glossary

When we rewrite expressions in factored form, it is helpful to remember that:

- Multiplying two positive numbers or two negative numbers results in a positive product.
- Multiplying a positive number and a negative number results in a negative product.

This means that if we want to find two factors whose product is 10, the factors must be both positive or both negative. If we want to find two factors whose product is -10, one of the factors must be positive and the other negative.

Suppose we wanted to rewrite $x^2 - 8x + 7$ in factored form. Recall that subtracting a number can be thought of as adding the opposite of that number, so that expression can also be written as $x^2 + -8x + 7$. We are looking for two numbers that:

- Have a product of 7. The candidates are 7 and 1, and -7 and -1.
- Have a sum of -8. Only -7 and -1 from the list of candidates meet this condition.

The factored form of $x^2 - 8x + 7$ is therefore $(x + -7)(x + -1)$ or, written another way, $(x - 7)(x - 1)$.

To write $x^2 + 6x - 7$ in factored form, we would need two numbers that:

- Multiply to make -7. The candidates are 7 and -1, and -7 and 1.
- Add up to 6. Only 7 and -1 from the list of candidates add up to 6.

The factored form of $x^2 + 6x - 7$ is $(x + 7)(x - 1)$.

Unit 7 Lesson 22 Practice Problems

1. Find two numbers that do the following. If you get stuck, try listing all the factors of the first number.
    a. multiply to -40 and add to -6
    b. multiply to -40 and add to 6
    c. multiply to -36 and add to 9
    d. multiply to -36 and add to -5

2. Create a diagram to show that $(x-5)(x+8)$ is equivalent to $x^2+3x-40$.

3. Write a + or a − sign in each box so the expressions on each side of the equal sign are equivalent.
    a. $(x\boxed{}18)(x\boxed{}3) = x^2 - 15x - 54$
    b. $(x\boxed{}18)(x\boxed{}3) = x^2 + 21x + 54$
    c. $(x\boxed{}18)(x\boxed{}3) = x^2 + 15x - 54$
    d. $(x\boxed{}18)(x\boxed{}3) = x^2 - 21x + 54$

4. Match each quadratic expression in standard form with its equivalent expression in factored form.

   a. $x^2 - 2x - 35$
   b. $x^2 + 12x + 35$
   c. $x^2 + 2x - 35$
   d. $x^2 - 12x + 35$

   1. $(x+5)(x+7)$
   2. $(x-5)(x-7)$
   3. $(x+5)(x-7)$
   4. $(x-5)(x+7)$

5. Rewrite each expression in factored form. If you get stuck, try drawing a diagram.

   a. $x^2 - 3x - 28$
   b. $x^2 + 3x - 28$
   c. $x^2 + 12x - 28$
   d. $x^2 - 28x - 60$

6. Which equation has exactly one solution?

   a. $x^2 = -4$
   b. $(x+5)^2 = 0$
   c. $(x+5)(x-5) = 0$
   d. $(x+5)^2 = 36$

(From Unit 7, Lesson 20)

7. Elena solves the equation $x^2 = 7x$ by dividing both sides by x to get $x = 7$. She says the solution is 7.

   Lin solves the equation $x^2 = 7x$ by rewriting the equation to get $x^2 - 7x = 0$. When she graphs the equation $y = x^2 - 7x$, the x-intercepts are $(0,0)$ and $(7,0)$. She says the solutions are 0 and 7.

   Do you agree with either of them? Explain or show how you know.

   (From Unit 7, Lesson 20)

8. Add or subtract:

   a. $(4x^2 + 3x + 7) + (8x^2 - 5x - 2)$

   b. $(m^2 - 9) + (3m^2 - 4m + 16)$

   c. $(7.2h^2 + 3h - 3.5) - (2.4h^2 - 5h + 1)$

   (From Unit 7, Lessons 14 and 15)

9. A bacteria population, p, can be represented by the equation $p = 100,000 \cdot (\frac{1}{4})^d$, where d is the number of days since it was measured.

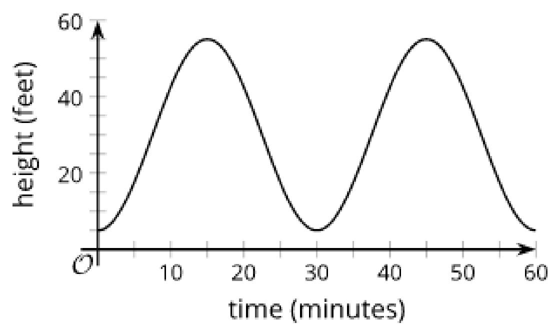
   a. What was the population 3 days before it was measured? Explain how you know.

   b. What is the last day when the population was more than 1,000,000? Explain how you know.

(From Unit 6)

10. The graph represents function $H(t)$, defined as the height of a passenger car on a ferris wheel, in feet, as a function of time, in seconds.

    Use the graph to help you:



   a. Find $H(0)$.

   b. Does $H(t) = 0$ have a solution? Explain how you know.

   c. Describe the domain of the function.

   d. Describe the range of the function.

(From Unit 5)

Lesson 23: Rewriting Quadratic Expressions in Factored Form (Part Three)

Learning Targets

- I can describe and justify the structure of the results when multiplying a sum and a difference, $(x+m)(x-m)$.
- When given quadratic expressions in the form of x^2+bx+c, I can rewrite them in factored form.

Bridge

Diego and Noah are trying to remember how to simplify $7x+-7x$. Diego thinks the answer is x, and Noah thinks the answer is 0. Do you agree with Diego, Noah, both, or neither? Explain your answer.

Warm-up: Products of Large-ish Numbers

Find each product mentally.

1. $9 \cdot 11$
2. $19 \cdot 21$
3. $99 \cdot 101$
4. $109 \cdot 101$

Activity 1: Can Products Be Written as Differences?

1. Clare claims that $(10+3)(10-3)$ is equivalent to 10^2-3^2 and $(20+1)(20-1)$ is equivalent to 20^2-1^2. Do you agree? Show your reasoning.

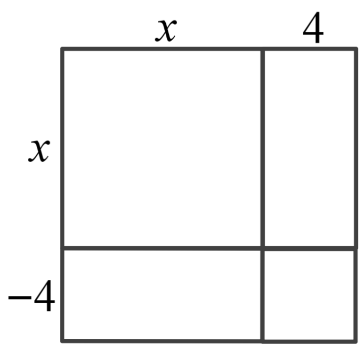
2.
   a. Use your observations from the first question and evaluate $(100+5)(100-5)$. Show your reasoning.

   b. Check your answer by computing $105 \cdot 95$.

3. Is $(x+4)(x-4)$ equivalent to $x^2 - 4^2$?

   Support your answer:

   a. With a diagram

   b. Without a diagram

4. Is $(x+4)^2$ equivalent to $x^2 + 4^2$? Support your answer, either with or without a diagram.

Are You Ready For More?

1. Explain how your work in the previous questions can help you mentally evaluate $22 \cdot 18$ and $45 \cdot 35$.

2. Here is a shortcut that can be used to mentally square any two-digit number. Let's take 83^2, for example.

   - 83 is $80 + 3$.
   - Compute 80^2 and 3^2, which give 6,400 and 9. Add these values to get 6,409.
   - Compute $80 \cdot 3$, which is 240. Double it to get 480.
   - Add 6,409 and 480 to get 6,889.

   Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

Activity 2: What If There Is No Linear Term?

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram. (Heads up: one of them is impossible.)

Partner	Factored form	Standard form
Partner A	$(x-10)(x+10)$	
Partner B	$(2x+1)(2x-1)$	
Partner A	$(4-x)(4+x)$	
Partner B		x^2-81
Partner A		$49-y^2$
Partner B		$9z^2-16$
Partner A		$25t^2-81$
Partner B	$(c+\frac{2}{5})(c-\frac{2}{5})$	
Partner A		$\frac{49}{16}-d^2$
Partner B	$(x+5)(x+5)$	
Partner A		x^2-6
Partner B		x^2+100

Lesson Debrief

Lesson 23 Summary and Glossary

Sometimes expressions in standard form don't have a linear term. Can they still be written in factored form?

Let's take $x^2 - 9$ as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0: $x^2 + 0x - 9$. (The expression $x^2 - 0x - 9$ is equivalent to $x^2 - 9$ because 0 times any number is 0, so $0x$ is 0.)

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is $(x+3)(x-3)$.

To check that this expression is indeed equivalent to $x^2 - 9$, we can expand the factored expression by applying the distributive property: $(x+3)(x-3) = x^2 - 3x + 3x + (-9)$. Adding $-3x$ and $3x$ gives 0, so the expanded expression is $x^2 - 9$.

In general, a quadratic expression that is a difference of two squares and has the form $a^2 - b^2$ can be rewritten as $(a+b)(a-b)$.

Here is a more complicated example: $49 - 16y^2$. This expression can be written as $7^2 - (4y)^2$, so an equivalent expression in factored form is $(7+4y)(7-4y)$.

What about $x^2 + 9$? Can it be written in factored form?

Let's think about this expression as $x^2 + 0x + 9$. Can we find two numbers that multiply to make 9 but add up to 0? Here are factors of 9 and their sums:

- 9 and 1, sum: 10
- -9 and -1, sum: -10
- 3 and 3, sum: 6
- -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write $x^2 + 9$ in factored form using the kinds of numbers that we know about.

Unit 7 Lesson 23 Practice Problems

1. Match each quadratic expression given in factored form with an equivalent expression in standard form. One expression in standard form has no match.

   a. $(y+x)(y-x)$

   b. $(11+x)(11-x)$

   c. $(x-11)(x+11)$

   d. $(x-y)(x-y)$

   1. $121-x^2$

   2. $x^2+2xy-y^2$

   3. y^2-x^2

   4. $x^2-2xy+y^2$

   5. x^2-121

2. Both $(x-3)(x+3)$ and $(3-x)(3+x)$ contain a sum and a difference and have only 3 and x in each factor.

   If each expression is rewritten in standard form, will the two expressions be the same? Explain or show your reasoning.

3.

a. Show that the expressions $(5+1)(5-1)$ and 5^2-1^2 are equivalent.

b. The expressions $(30-2)(30+2)$ and 30^2-2^2 are equivalent and can help us find the product of two numbers. Which two numbers are they?

c. Write $94 \cdot 106$ as a product of a sum and a difference and then as a difference of two squares. What is the value of $94 \cdot 106$?

4. Write each expression in factored form. If not possible, write "not possible."

a. x^2-144

b. x^2+16

c. $25-x^2$

d. b^2-a^2

e. $100+y^2$

5. Create a diagram to show that $(x-3)(x-7)$ is equivalent to $x^2-10x+21$.

(From Unit 7, Lesson 21)

6. Select **all** the expressions that are equivalent to $8-x$.

   a. $x-8$

   b. $8+(-x)$

   c. $-x-(-8)$

   d. $-8+x$

   e. $x-(-8)$

   f. $x+(-8)$

   g. $-x+8$

(From Unit 7, Lesson 21)

7. What are the solutions to the equation $(x-a)(x+b)=0$?

   a. a and b

   b. $-a$ and $-b$

   c. a and $-b$

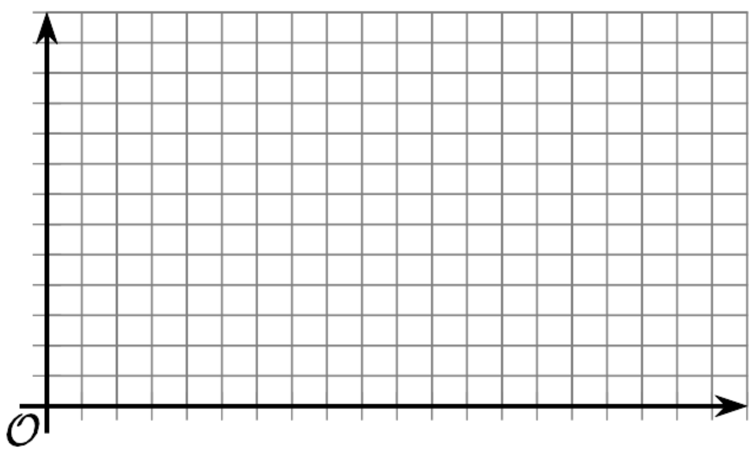
   d. $-a$ and b

(From Unit 7, Lesson 19)

8. The function f, defined by $f(t) = 1,000 \cdot (1.07)^t$, represents the amount of money in a bank account t years after it was opened.

   a. How much money was in the account when it was opened?

   b. Sketch a graph of f.



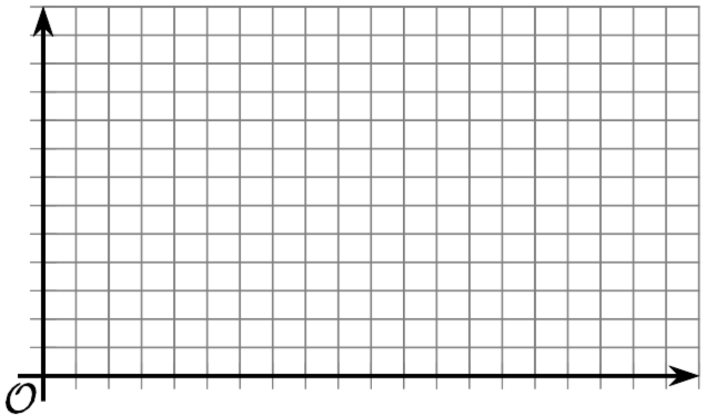
   c. When does the account value reach $2,000?

(From Unit 6)

9. Mai fills a tall cup with hot cocoa, 12 centimeters in height. She waits 5 minutes for it to cool. Then, she starts drinking in sips, at an average rate of 2 centimeters of height every 2 minutes, until the cup is empty.

   The function C gives the height of hot cocoa in Mai's cup, in centimeters, as a function of time, in minutes.v

   a. Sketch a possible graph of C. Be sure to include a label and a scale for each axis.



   b. What quantities do the domain and range represent in this situation?

   c. Describe the domain and range of C.

(From Unit 5)

10. Select all expressions equivalent to $11 - 5x$.

    a. $6x + 11 - x$

    b. $6 - 6x + 5 + x$

    c. $-2(3x - 5) + (x + 1)$

    d. $2(3x + 5) - (11x - 1)$

(Addressing NC.7.EE.1)

Lesson 24: Solving Quadratic Equations by Using Factored Form

Learning Targets

- I can recognize quadratic equations that have 0, 1, or 2 real solutions when they are written in factored form.
- I can solve a quadratic equation using factored form.

Bridge

For each equation, identify if there are no solutions, one solution, or infinitely many solutions.

1. $5x + 7 = 5x + 7$
2. $3(x + 4) = 3x + 11$
3. $2x - 6 = 8x$

Warm-up: Why Would You Do That?

Let's try to find at least one solution to $x^2 - 2x - 35 = 0$.

1. Choose a whole number between 0 and 10.

2. Evaluate the expression $x^2 - 2x - 35$, using your number for x.

3. If your number doesn't give a value of 0, look for someone in your class who may have chosen a number that does make the expression equal 0. Which number is it?

4. There is another number that would make the expression $x^2 - 2x - 35$ equal 0. Can you find it?

Activity 1: Let's Solve Some Equations!

1. To solve the equation $n^2 - 2n = 99$, Tyler wrote out the following steps. Analyze Tyler's work. Write down what they did in each step.

$n^2 - 2n = 99$	Original equation
$n^2 - 2n - 99 = 0$	Step 1
$(n - 11)(n + 9) = 0$	Step 2
$n - 11 = 0$ or $n + 9 = 0$	Step 3
$n = 11$ or $n = -9$	Step 4

2. Solve each equation by rewriting it in factored form and using the zero product property. Show your reasoning.

   a. $x^2 + 8x + 15 = 0$

   b. $x^2 - 8x + 12 = 5$

   c. $x^2 - 10x - 11 = 0$

   d. $49 - x^2 = 0$

   e. $(x+4)(x+5) - 30 = 0$

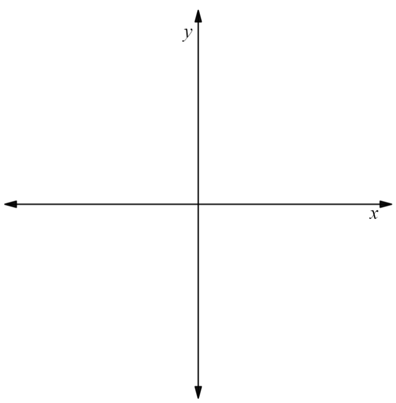
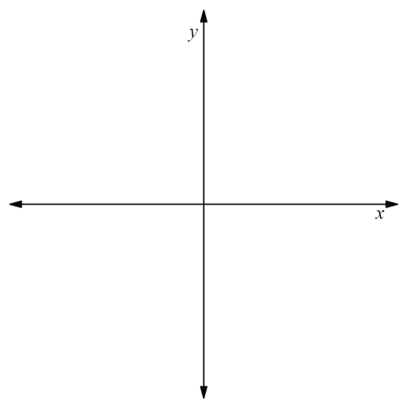
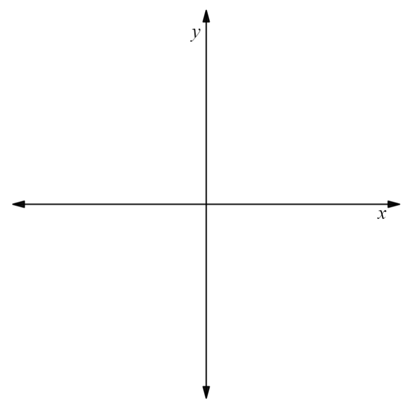
Are You Ready For More?

Solve this equation and explain or show your reasoning.

$(x^2 - x - 20)(x^2 + 2x - 3) = (x^2 + 2x - 8)(x^2 - 8x + 15)$

Activity 2: Revisiting Quadratic Equations with Only One Solution

1. The other day, we saw that a quadratic equation can have 0, 1, or 2 solutions. Sketch graphs that represent three quadratic functions: one that has no zeros, one with 1 zero, and one with 2 zeros.

No zeros	One zero	Two zeros
		

2. Use graphing technology to graph the function defined by $f(x) = x^2 - 2x + 1$. What do you notice about the x-intercepts of the graph? What do the x-intercepts reveal about the function?

3. Solve $x^2 - 2x + 1 = 0$ by using the factored form and zero product property. Show your reasoning. What solutions do you get?

4. Write an equation to represent another quadratic function that you think will only have one zero. Graph it to check your prediction.

Lesson Debrief

Lesson 24 Summary and Glossary

Recently, we learned strategies for transforming expressions from standard form to factored form. In earlier lessons, we have also seen that when a quadratic expression is in factored form, it is pretty easy to find values of the variable that make the expression equal zero. Suppose we are solving the equation $x(x+4)=0$, which says that the product of x and $x+4$ is 0. By the zero product property, we know this means that either $x=0$ or $x+4=0$, which then tells us that 0 and -4 are solutions.

Together, these two skills—writing quadratic expressions in factored form and using the zero product property when a factored expression equals 0—allow us to solve quadratic equations given in other forms. Here is an example:

$n^2-4n=140$	Original equation
$n^2-4n-140=0$	Subtract 140 from each side so the right side is 0
$(n-14)(n+10)=0$	Rewrite in factored form
$n-14=0$ or $n+10=0$	Apply the zero product property
$n=14$ or $n=-10$	Solve each equation

When a quadratic equation is written as $expression\ in\ factored\ form=0$, we can also see the number of solutions the equation has.

In the example earlier, it was not obvious how many solutions there would be when the equation was $n^2-4n-140=0$. When the equation was rewritten as $(n-14)(n+10)=0$, we could see that there were two numbers that could make the expression equal 0: 14 and -10.

How many solutions does the equation $x^2-20x+100=0$ have?

Let's rewrite it in factored form: $(x-10)(x-10)=0$. The two factors are identical, which means that there is only one value of x that makes the expression $(x-10)(x-10)$ equal 0. The equation has only one solution: 10.

Unit 7 Lesson 24 Practice Problems

1. Find **all** the solutions to each equation.

   a. $x(x-1)=0$

   b. $(5-x)(5+x)=0$

   c. $(2x+1)(x+8)=0$

   d. $(3x-3)(3x-3)=0$

   e. $(7-x)(x+4)=0$

2. Rewrite each equation in factored form and solve using the zero product property.

   a. $d^2 - 7d + 6 = 0$

   b. $x^2 + 18x + 81 = 0$

   c. $u^2 + 7u - 60 = 0$

   d. $x^2 + 0.2x + 0.01 = 0$

3. Here is how Elena solves the quadratic equation $x^2 - 3x - 18 = 0$.

$$\begin{aligned} x^2 - 3x - 18 &= 0 \\ (x-3)(x+6) &= 0 \\ x - 3 = 0 \ \ or \ \ x + 6 &= 0 \\ x = 3 \ \ or \ \ x &= -6 \end{aligned}$$

   Is her work correct? If you think there is an error, explain the error and correct it.

   Otherwise, check her solutions by substituting them into the original equation and showing that the equation remains true.

4. Tyler is working on solving a quadratic equation, as shown here.

$$\begin{aligned} p^2 - 5p &= 0 \\ p(p-5) &= 0 \\ p - 5 &= 0 \\ p &= 5 \end{aligned}$$

They think that their solution is correct because substituting 5 for p in the original expression $p^2 - 5p$ gives $5^2 - 5(5)$, which is $25 - 25$ or 0.

Explain the mistake that Tyler made and show the correct solutions.

5. Which expression is equivalent to $x^2 - 7x + 12$?

   a. $(x+3)(x+4)$

   b. $(x-3)(x-4)$

   c. $(x+2)(x+6)$

   d. $(x-2)(x-6)$

(From Unit 7, Lesson 21)

6. These quadratic expressions are given in standard form. Rewrite each expression in factored form. If you get stuck, try drawing a diagram.

   a. $x^2 + 7x + 6$

   b. $x^2 - 7x + 6$

   c. $x^2 - 5x + 6$

   d. $x^2 + 5x + 6$

(From Unit 7, Lesson 21)

7. Choose a statement to correctly describe the zero product property.

   If a and b are numbers, and $a \cdot b = 0$, then:

   a. Both a and b must equal 0.

   b. Neither a nor b can equal 0.

   c. Either $a = 0$ or $b = 0$.

   d. $a + b$ must equal 0.

(From Unit 7, Lesson 19)

8. Select **all** the functions whose output values will eventually overtake the output values of function f defined by $f(x) = 25x^2$.

   a. $g(x) = 5(2)^x$

   b. $h(x) = 5^x$

   c. $j(x) = x^2 + 5$

   d. $k(x) = (\frac{5}{2})^x$

   e. $n(x) = 2x^2 + 5$

(From Unit 7, Lesson 3)

9. (*Technology required*.) A moth population, p, is modeled by the equation $p = 500,000 \cdot (\frac{1}{2})^w$, where w is the number of weeks since the population was first measured.

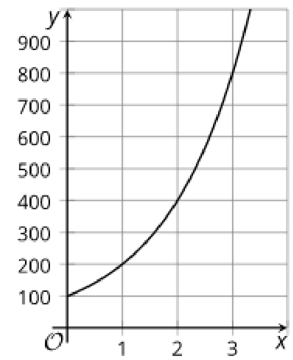
   a. What was the moth population when it was first measured?

   b. What was the moth population after 1 week? What about 1.5 weeks?

   c. Use technology to graph the population and find out when it falls below 10,000.

(From Unit 6)

10. Here is a graph of the function f given by $f(x) = 100 \cdot 2^x$.



Suppose g is the function given by $g(x) = 50 \cdot (1.5)^x$.

Will the graph of g meet the graph of f for any positive value of x? Explain how you know.

(From Unit 6)

11. For each equation, identify if there are no solutions, one solution, or infinitely many solutions.

a. $3x + 9 = 3(x + 9)$

b. $2(x + 4) = 2(x + 2) + 4$

c. $5x - 4 = 8x - 4$

(Addressing NC.8.EE.7)

Lesson 25: Rewriting Quadratic Expressions in Factored Form (Part Four)

Learning Targets

- When given quadratic expressions of the form $ax^2 + bx + c$, and a is not 1, I can write equivalent expressions in factored form.
- I can use the factored form of a quadratic expression to answer questions about a situation.

Bridge

A “factor pair” for a given number is a pair of whole numbers that, when multiplied, result in that number. For example, 5 and 4 are a factor pair of 20 because $5 \cdot 4 = 20$.

Find all factor pairs of each of the following numbers.

1. 12
2. 36
3. 5

Warm-up: Quadratic Expressions

Which one doesn’t belong? Explain your reasoning.

a. $(x+4)(x-3)$	b. $3x^2 - 8x + 5$
c. $x^2 - 25$	d. $x^2 + 2x + 3$

Activity 1: A Little More Advanced

Complete the tables so that each row has a pair of equivalent expressions. If you get stuck, try drawing a diagram.

1.

Factored form	Standard form
$(3x+1)(x+4)$	
$(3x+2)(x+2)$	
$(3x+4)(x+1)$	

2.

Factored form	Standard form
	$5x^2+21x+4$
	$3x^2+15x+12$
	$6x^2+19x+10$

Are You Ready For More?

Here are three quadratic equations, each with two solutions. Find both solutions to each equation, using the zero product property somewhere along the way. Show each step in your reasoning.

1. $x^2 = 6x$
2. $x(x+4) = x+4$
3. $2x(x-1) + 3x - 3 = 0$

Activity 2: Timing A Blob of Water

An engineer is designing a fountain that shoots out drops of water. The nozzle from which the water is launched is 2 meters above the ground. It shoots out a drop of water at a vertical velocity of 9 meters per second.

Function h models the height in meters, h, of a drop of water t seconds after it is shot out from the nozzle. The function is defined by the equation $h(t) = -5t^2 + 9t + 2$.

How many seconds until the drop of water hits the ground?

1. Write an equation that we could solve to answer the question.

2. Solve the equation by writing the expression in factored form and using the zero product property.

3. What is the answer to the original question? Explain how it is related to the solutions to the equation.

Activity 3: Making It Simpler

1. Here are two strategies for expanding $(2x+3)(4x+5)$ to write the equivalent expression $8x^2+22x+15$.

Expanding using the distributive property

$$2x(4x+5)+3(4x+5)$$
$$8x^2+10x+12x+15$$
$$8x^2+22x+15$$

Expanding using a diagram

	$2x$	3
$4x$	$8x^2$	$12x$
5	$10x$	15

$$8x^2+22x+15$$

a. Identify the two terms that add to $22x$ and find the product of the coefficients.

b. Find the product of the coefficients of the first term $8x^2$ and the last term 15.

c. What do you notice?

2. Expand each of the following and check if what you noticed in question 1 holds true.

a. $(3x-4)(x+7)$

b. $(5x-1)(2x-3)$

c. $(px+m)(qx+n)$

3. Consider how this pattern can help in factoring the expression $6x^2 + 19x + 10$.

   a. Find the product of $a \cdot c$.

   b. Find the factors of $a \cdot c$ that add to make b.

   c. Use the factors to fill in the missing information and finish writing the expression in factored form.

Using the distributive property

$6x^2+$______$x+$______$x + 10$

Using a diagram

$6x^2$	____x
____x	10

4. Try this method to write each of these in factored form.

   a. $6x^2 + 17x + 12$

   b. $5x^2 - 17x + 6$

Lesson Debrief

Lesson 25 Summary and Glossary

In some cases, rewriting an expression of the form $ax^2 + bx + c = 0$ in factored form is challenging.

For example, what is the factored form of $6x^2 + 11x - 35$?

We know that it could be $(3x + \square)(2x + \square)$, or $(6x + \square)(x + \square)$, but will the second number in each factor be -5 and 7, 5 and -7, 35 and -1, or -35 and 1? And in which order?

We have to do some guessing and checking before finding the equivalent expression that would allow us to solve the equation $6x^2 + 11x - 35 = 0$.

Once we find the right factors, we can proceed to solving using the zero product property, as shown here:

$$
\begin{gathered}
6x^2 + 11x - 35 = 0 \\
(3x - 5)(2x + 7) = 0 \\
3x - 5 = 0 \;\; or \;\; 2x + 7 = 0 \\
n = \tfrac{5}{3} \;\; or \;\; x = -\tfrac{7}{2}
\end{gathered}
$$

Rewriting quadratics into factored form and using the zero product property has its limitations. Sometimes it can be challenging to factor, especially when a is not 1. Other times the quadratic cannot be factored. In those cases we can use a graph to approximate solutions. Other methods for solving quadratics will be explored in future courses.

Unit 7 Lesson 25 Practice Problems

1. To write $11x^2 + 17x - 10$ in factored form, Diego first listed pairs of factors of -10.

$$(\_\_ + 5)(\_\_ + -2)$$
$$(\_\_ + 2)(\_\_ + -5)$$
$$(\_\_ + 10)(\_\_ + -1)$$
$$(\_\_ + 1)(\_\_ + -10)$$

   a. Use what Diego started to complete the rewriting.

   b. How did you know you'd found the right pair of expressions? What did you look for when trying out different possibilities?

2. To rewrite $4x^2 - 12x - 7$ in factored form, Jada listed some pairs of factors of $4x^2$:

$$(2x + \_\_)(2x + \_\_)$$
$$(4x + \_\_)(1x + \_\_)$$

   Use what Jada started to rewrite $4x^2 - 12x - 7$ in factored form.

3. Rewrite each quadratic expression in factored form. Then, use the zero product property to solve the equation.

   a. $7x^2 - 22x + 3 = 0$

   b. $4x^2 + x - 5 = 0$

   c. $9x^2 - 25 = 0$

4. Han is solving the equation $5x^2 + 13x - 6 = 0$.

   Here is his work:

   $$5x^2 + 13x - 6 = 0$$
   $$(5x - 2)(x + 3) = 0$$
   $$x = 2 \text{ or } x = -3$$

   Describe Han's mistake. Then, find the correct solutions to the equation.

5. Which equation shows a next step in solving $9(x-1)^2=36$ that will lead to the correct solutions?

    a. $9(x-1)=6 \quad or \quad 9(x-1)=-6$

    b. $3(x-1)=6$

    c. $(x-1)^2=4$

    d. $(9x-9)^2=36$

(From Unit 7, Lesson 18)

6. To solve the equation $0=4x^2-28x+39$, Elena uses technology to graph the function $f(x)=4x^2-28x+39$. She finds that the graph crosses the x-axis at $(1.919, 0)$ and $(5.081, 0)$.

    a. What is the name for the points where the graph of a function crosses the x-axis?

    b. Use a calculator to compute $f(1.919)$ and $f(5.081)$.

    c. Explain why 1.919 and 5.081 are approximate solutions to the equation $0=4x^2-28x+39$ and are not exact solutions.

(From Unit 7, Lesson 17)

7. A picture is 10 inches wide by 15 inches long. The area of the picture, including a frame that is x inch(es) thick, can be modeled by the function $A(x) = (2x + 10)(2x + 15)$.

   a. Use function notation to write a statement that means: the area of the picture, including a frame that is 2 inches thick, is 266 square inches.

   b. What is the total area if the picture has a frame that is 4 inches thick?

(From Unit 7, Lesson 16)

8. Add or subtract:

   a. $(5x^2 - 4x + 3) - (2x^2 - 4x - 7)$

   b. $(9 - 7x^2) + (3x^2 - 3.4x + 5.2)$

   c. $(\frac{3}{4}x^2 - 4x) + (\frac{3}{4}x^2 - 4x + \frac{5}{8})$

(From Unit 7, Lessons 14 and 15)

9. (*Technology required.*) The number of people, p, who watch a weekly TV show is modeled by the equation $p = 100,000 \cdot (1.1)^w$, where w is the number of weeks since the show first aired.

    a. How many people watched the show the first time it aired? Explain how you know.

    b. Use technology to graph the equation.

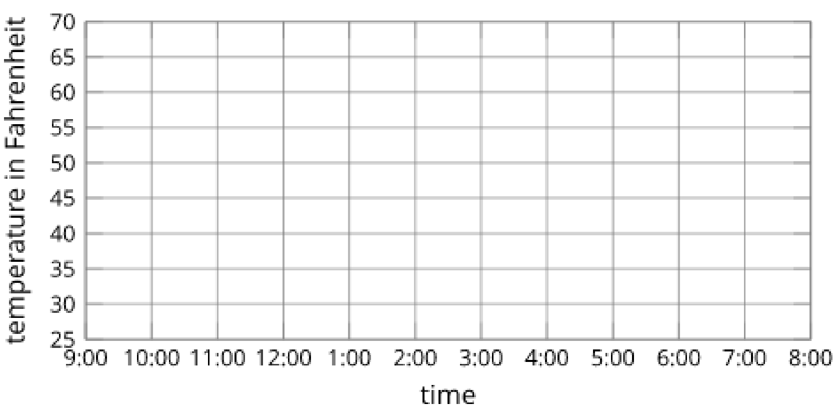
    c. In which week does the show first get an audience of more than 500,000 people?

(From Unit 6)

10. Here is a description of the temperature at a certain location yesterday.

    "It started out cool in the morning, but then the temperature increased until noon. It stayed the same for a while until it suddenly dropped quickly! It got colder than it was in the morning, and after that, it was cold for the rest of the day."

    Sketch a graph of the temperature as a function of time.



(From Unit 5)

Lesson 26: Factor to Identify Key Features and Solve Equations

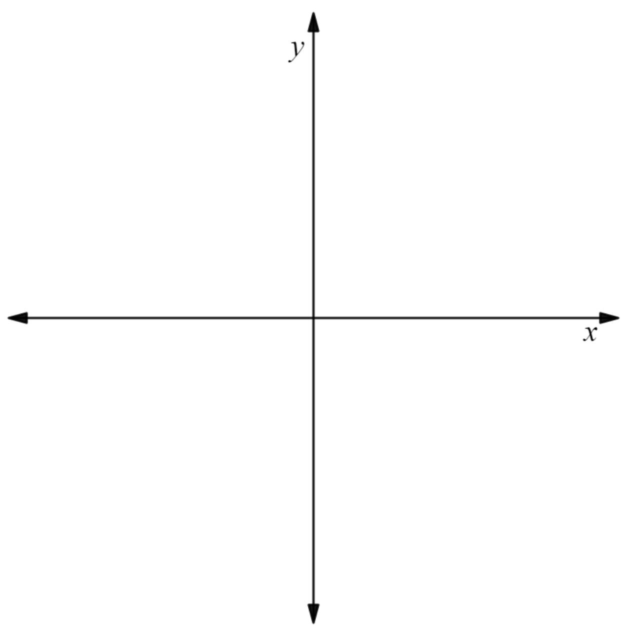
Learning Targets

- I can factor a quadratic function of the form $f(x) = ax^2 + bx + c$ in order to identify and explain key features of the function.
- I can factor and solve a quadratic equation of the form $ax^2 + bx + c = 0$.

Warm-up: Sketching the Graph of a Quadratic Function

Without using technology, sketch the graph of $f(x) = (2x+3)(x-4)$ by finding the:

1. x-intercepts
2. y- intercept
3. vertex



Activity 1: The Grasshopper's Jump

A grasshopper is sitting on a flower and jumps to the ground. The quadratic function $h(t) = -5t^2 + t + 6$ models the grasshopper's height above the ground, in inches, as a function of time, t, in seconds.

1. Write the function in factored form.

2. Use the factored form or the standard form to answer each of the following questions.

   a. At what height was the grasshopper when it was sitting on the flower? Explain how you know.

   b. How many seconds after jumping off the flower did the grasshopper land on the ground? Explain how you know.

   c. What was the maximum height above the ground of the grasshopper?

Activity 2: Solve by Factoring

1. Han solved the equation $6x^2 + 11x - 10 = 0$ by factoring. His work is shown below.

   $6x^2 + 11x - 10 = 0$
   $(2x - 5)(3x + 2) = 0$
   $2x - 5 = 0$ or $3x + 2 = 0$
   $x = \frac{5}{2}$ or $x = \frac{-2}{3}$

   Are Han's solutions correct? If the solutions are correct, explain how you know. If the solutions are incorrect, identify the error.

2. Mai is solving the equation $4x^2 + 12x + 5 = 0$. She is using guess and check to factor the expression $4x^2 + 12x + 5$. So far she has tried $(4x + 1)(x + 5)$ and $(4x + 5)(x + 1)$, and neither of them has worked. She is not sure what to do now. What suggestion would you give Mai that would help her factor the expression?

3. Kiran is solving the equation $5x^2 - 12x + 11 = 4$. He is using guess and check to factor the expression $5x^2 - 12x + 11$. So far he has tried $(5x - 1)(x - 11)$ and $(5x - 11)(x - 1)$, and neither of them has worked. What suggestion would you give Kiran that would help him understand the error he has made?

Lesson Debrief

Lesson 26 Summary and Glossary

We have seen that when a quadratic expression is expressed in standard form, it can be beneficial to rewrite it in factored form. The factored form, along with the zero product property, allows us to solve quadratic equations and identify key features of quadratic functions.

Recently, we saw that factoring an expression of the form $ax^2 + bx + c$ when a is not 1 can be challenging; however, when factoring is possible, it gives us another way to reason with quadratics.

Here is an example of factoring a quadratic expression to identify key features of a function:

- A diver's height above the water, in meters, as a function of time t, in seconds, since jumping from the diving board can be modeled by the function $h(t) = -5t^2 + 2t + 3$.
- The coefficient a is -5. We will need to try several possibilities to factor the expression. These include:
  - $(-5t + 3)(t + 1) = -5t^2 - 2t + 3$
  - $(-5t - 3)(t - 1) = -5t^2 + 2t + 3$ ← This is the equivalent factored form.
- Using the factored form and the zero product property we can determine that:
  - The zeros of the function are $t = \frac{-3}{5}$ and $t = 1$.
  - The horizontal intercepts are $(\frac{-3}{5}, 0)$ and $(1, 0)$.
- This means that the diver hits the water 1 second after jumping from the diving board.
- Using the horizontal intercepts, we can determine that the maximum height occurs when $t = 0.2$. Since $h(0.2) = 3.2$, the maximum height of the diver is 3.2 meters. This occurs 0.2 seconds after jumping from the diving board.

Here is an example of factoring a quadratic expression to solve a quadratic equation.

- Given the equation $8x^2 - 2x - 21 = 0$. we notice that the quadratic expression is equal to zero and that the coefficient of the squared term is 8. To factor the expression we will need to try some possibilities. These include:
  - $(8x - 7)(x + 3) = 8x^2 + 17x - 21$
  - $(8x - 3)(x + 7) = 8x^2 + 53x - 21$
  - $(4x - 3)(2x + 7) = 8x^2 + 22x - 21$
  - $(4x - 7)(2x + 3) = 8x^2 - 2x - 21$ ← This is the equivalent factored form.
- We can apply the zero product property to solve $(4x - 7)(2x + 3) = 0$. Thus, the solutions are $x = \frac{7}{3}$ and $x = \frac{-3}{2}$.

Rewriting a quadratic expression to identify key features of a quadratic function or to solve a quadratic equation can be useful especially when used with the zero product property. It is limited, however, to only those expressions that can be factored.

Unit 7 Lesson 26 Practice Problems

1. Jada is solving the equation $9x^2 - 34x - 8 = 0$. She is using guess and check to factor the expression $9x^2 - 34x - 8$. She found that $(9x-2)(x+4) = 9x^2 + 34x - 8$, but she doesn't have it yet because the expression should have a $-34x$.

    a. What suggestion would you give Jada that would help her factor the expression?

    b. Find the solutions to the equation.

2. The function $A(w) = -2w^2 + 25w - 33$ represents the area of a rectangle, in square feet, as a function of the width, w, in feet. Factor the quadratic expression and use the factored form to determine the maximum area of the rectangle and the width of that rectangle.

3. Rewrite each quadratic expression in standard form.

   a. $(x+1)(7x+2)$

   b. $(8x+1)(x-5)$

   c. $(2x+1)(2x-1)$

   d. $(4+x)(3x-2)$

(From Unit 7, Lesson 25)

4. Find the missing expression in parentheses so that each pair of quadratic expressions is equivalent. Show that your expression meets this requirement.

   a. $(4x-1)(\underline{\qquad})$ and $16x^2-8x+1$

   b. $(9x+2)(\underline{\qquad})$ and $9x^2-16x-4$

   c. $(\underline{\qquad})(-x+5)$ and $-7x^2+36x-5$

(From Unit 7, Lesson 25)

5. (*Technology required.*) For each equation, find the approximate solutions by graphing.

   a. $x^2 + 10x + 8 = 0$

   b. $x^2 - 4x - 11 = 0$

(From Unit 7, Lesson 25)

6. Match an equation in the first column with an equivalent equation in the second column.

   a. $y = 3x - 5$
   b. $(x+1)(x-1) = 0$
   c. $4t(t-2) = 10$
   d. $x^2 + 2x + 1 = 0$
   e. $4t + 16 = 10$
   f. $3x = 5y$

   1. $4(t+4) = 10$
   2. $y - 3x + 5 = 0$
   3. $y = \frac{3x}{5}$
   4. $x^2 - 1 = 0$
   5. $4t^2 - 8t = 10$
   6. $(x+1)(x+1) = 0$

(From Unit 7, Lesson 25)

7. Solve each equation.

a. $p^2 + 10 = 7p$

b. $x^2 + 11x + 27 = 3$

c. $(y+2)(y+6) = -3$

(From Unit 7, Lesson 24)

8. Match the expressions in factored form with the expressions in standard form.

Expressions in factored form

a. $(2a+5)(a+4)$

b. $(3a-1)(a-10)$

c. $(a+7)(5a-2)$

d. $(4a-5)(4a-5)$

e. $(4a-5)(4a+5)$

f. $(2a+7)(9a+4)$

Functions in standard form

1. $f(x) = 2a^2 + 13a + 20$

2. $g(x) = 16a^2 - 25$

3. $h(x) = 5a^2 + 33a - 14$

4. $j(x) = 16a^2 - 40a + 25$

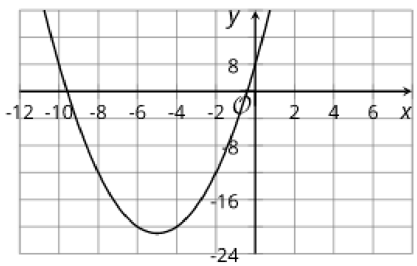
5. $k(x) = 18a^2 + 71a + 28$

6. $m(x) = 3a^2 - 31a + 10$

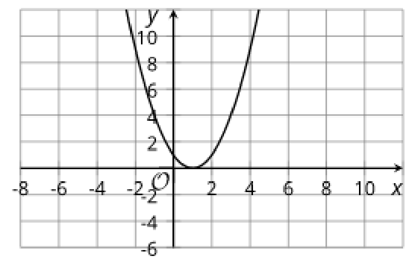
(From Unit 7, Lesson 21)

9. For each function f, decide if the equation $f(x) = 0$ has 0, 1, or 2 solutions. Explain how you know.

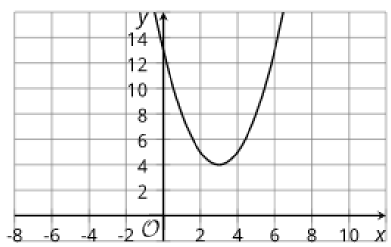
a.



b.



c.



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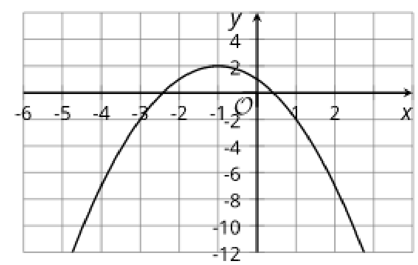

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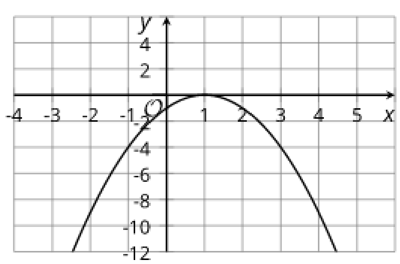
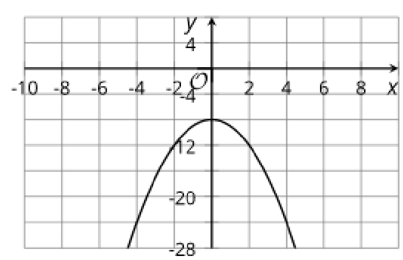


f.



(From Unit 7, Lesson 20)

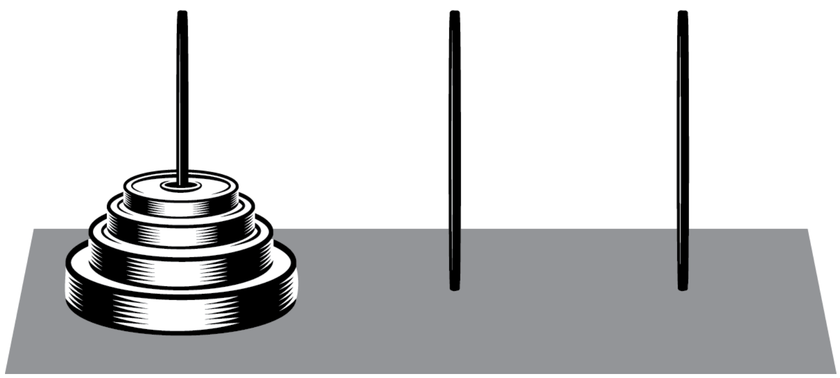
Lesson 27: Post-Test Activities

Learning Targets

- I can reflect on my progress in mathematics.
- I can share feedback that can help make me and my teacher grow.

Activity 2: The Tower of Hanoi[1]

In the Tower of Hanoi puzzle, a set of discs sits on a peg, while there are 2 other empty pegs.



A "move" in the Tower of Hanoi puzzle involves taking a disc and moving it to another peg. There are two rules:

- Only move 1 disc at a time.
- Never put a larger disc on top of a smaller one.

You complete the puzzle by building the complete tower on a different peg than the starting peg. As you answer the following questions, note the number of moves you've made in the table below with the goal of taking the smallest number of moves possible.

1. Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?

2. Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?

[1] Adapted from IM 9–12 Math https://curriculum.illustrativemathematics.org/HS/index.html,

3. Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.

4. How many moves do you think it will take to complete a puzzle with 5 discs? Explain or show your reasoning.

5. How many moves do you think it will take to complete a puzzle with 7 discs?

Number of discs	1	2	3	4	5	6	7
Number of moves							

Are You Ready For More?

A legend says that a Tower of Hanoi puzzle with 64 discs is being solved, one move per second. How long will it take to solve this puzzle? Explain how you know.